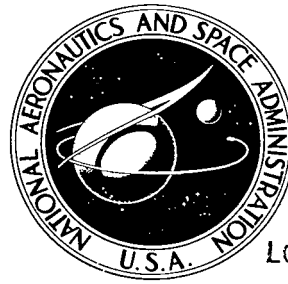


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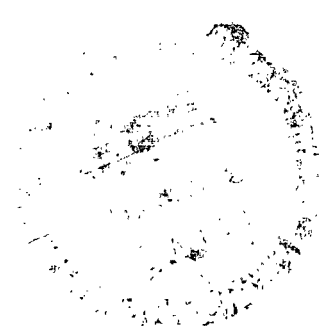
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# INVISCID HYPERSONIC FLOW OVER A BLUNT BODY WITH HIGH RATES OF MASS AND HEAT TRANSFER

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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# INVISCID HYPERSONIC FLOW OVER A BLUNT BODY WITH HIGH RATES OF MASS AND HEAT TRANSFER

By E. Dale Martin

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## SUMMARY

A study of steady hypersonic flow over a blunt body with a very high rate of continuous mass transfer from the body surface, with heat conduction, is presented. An approximate analysis is made of the inviscid flow over a sphere with large mass flux. Lighthill's well-known constant-density inviscid blunt-body flow solution was previously extended by Vinokur and Sanders and by Cresci and Libby to include inviscid flow at a high rate out of the body surface. The present analysis further extends Cresci and Libby's solution to include the effects of the boundary shock wave, a thin viscous region dominated by viscous-compressive stresses (rather than viscous shear) and the associated heat conduction in the gas adjacent to the body surface. These effects are important to consider when there is a significant source of translational nonequilibrium at the body surface (such as absorption of intense radiation and the accompanying large heat conduction).

## INTRODUCTION

Hypersonic flow over a blunt body with a high rate of mass transfer has been studied by a number of investigators. For example, in an early investigation, Hoshizaki (ref. 1) made numerical calculations from the incompressible Navier-Stokes equations to determine the effects of mass transfer on the heat-transfer rate to blunt bodies. Vinokur and Sanders (ref. 2) extended the constant-density inviscid solution of Lighthill (ref. 3) to include mass injection from the body. The calculations of reference 2 were made for the very special case wherein the injection is vectored so that the flow from the body is irrotational. Libby (ref. 4), Cresci and Libby (ref. 5), and Fox and Libby (ref. 6) also studied blunt-body flows with large mass-injection rates. Of particular interest here is the appendix of reference 5, which contains a solution to the inviscid, incompressible-flow equations that is equivalent (except for the vectored injection) to that given in reference 2. The inviscid solutions of Vinokur and Sanders and of Cresci and Libby are further discussed in references 7 and 8.

The magnitude of the mass flux from the body determines the nature of the interaction with the outer flow and the nature of the effects on the body motion and heating. When the mass flux is small enough, the outer flow is not affected appreciably, and boundary-layer theory is applicable if the Reynolds number  $\rho_2 V_2 d_0 / \mu_2$  is large. (Notation is defined in appendix A.) It will be shown in a later section that the bulk of the flow in the inner

region is essentially inviscid if the mass flux (or, more appropriately,  $Re_b$ ) is large; that a thick viscous region exists only for moderate values of mass flux; and that a boundary layer exists only for small mass flux (small  $Re_b$ ).

A thin region of flow dominated by viscous-compressive stresses ( $\tau_{rr}$ ) and heat conduction is expected to occur, in general, at a boundary surface from which a gas flows at a rapid rate with significant heat conduction at the surface (e.g., because of absorption of intense radiation there), as discussed in references 9 and 10. Thus, even though the bulk of the flow from the body is inviscid, a thin layer immediately adjacent to the body generally can have significant viscous effects which are quite different from the viscous-shearing effects (e.g., significant  $\tau_{r\theta}$ ) that characterize a boundary layer. In this region, designated as a boundary shock wave, the density, temperature, pressure, and velocity may vary rapidly. In the inviscid limit, the boundary shock wave is of vanishing thickness, and the flow variables change discontinuously across it (in the same sense as across a shock wave). Significant heat transfer at the wall and large  $Re_b$  are necessary and sufficient conditions for the presence of the boundary shock wave (ref. 10) whether the efflux is supersonic or subsonic. If the heat transfer is small, the effects of the boundary shock wave are negligible. If the efflux is supersonic relative to the surface ( $M_b > 1$ ), the limiting case of the boundary shock wave as the heat conduction approaches zero is a simple detached shock wave near the surface.

The effects of the boundary shock wave did not appear in the analyses of references 1, 2, 4, 5, and 6. Although the viscous terms were included in the incompressible form of the Navier-Stokes equations used in reference 1, the boundary-shock effects would not appear in a purely constant-density analysis unless discontinuities were allowed in order to account for rapid variations in density, normal velocity, etc. Similarly, the inviscid-flow analyses of reference 2 and the appendix of reference 5 contained neither viscous-compressive effects (effects of  $\tau_{rr}$ ) nor discontinuous flow variables to account for possible viscous effects at the body. Those analyses also would not apply to supersonic efflux because, with no heat conduction at the wall, there would have to be a detached shock wave somewhere between the body surface and the stagnation point. Boundary-shock effects did not appear in the boundary-layer analyses of references 4 and 6 because the terms in the Navier-Stokes equations that can be important very near the surface when the mass flow normal to the surface is large (terms containing  $\tau_{rr}$ ) are omitted in the boundary-layer equations used in those analyses. Since the boundary-shock effects are not important in some cases, the results of the previous investigations are valid and accurate (under appropriate conditions) even though account was not taken of a possible boundary shock wave.

It is hoped that the present study will shed some light on the nature of the division of the flow into inviscid and thin viscous regions. After justification of the approach used, the flow is analyzed by treating constant-density inviscid regions with lines of discontinuity to represent thin viscous regions. (Treating the internal structure of the viscous regions is beyond the scope of this report, but rigorous derivation of the boundary conditions for the inviscid regions, based on recognition of the presence of thin viscous regions, is contained in an appendix.) The analysis includes the effects of

the boundary shock wave, which makes the flow calculation correct in the extension to flows with more severe conditions than can be treated correctly by the previous analyses. The inviscid-flow solution is the same as that in the appendix of reference 5 except in the application of boundary conditions at the body surface. That is, the inviscid solution is the same as Cresci and Libby's if their boundary parameters are the same as values outside a boundary shock in the present analysis. Thus, one main purpose of this report is to relate the inviscid solution to appropriate boundary parameters under the conditions where the boundary shock wave is expected to occur according to the theory of references 9 and 10. The approach presented here may therefore be considered to be an extension of the analyses in reference 2 and the appendix of reference 5 to cases with significant heat conduction at the boundary surface due to a source of translational nonequilibrium, such as intense radiation being absorbed there.

#### DIVISION OF FLOW INTO INVISCID AND THIN VISCOUS REGIONS

The flow of air and of gas emanating from the body is assumed to be governed by the Navier-Stokes equations. They may be expressed symbolically as

$$\text{convective terms} = \frac{1}{\text{Re}} \cdot (\text{viscous terms}) \quad (1)$$

The convective terms and the viscous terms are made dimensionless with respect to the appropriate flow quantities in each region of interest so that the Reynolds number for the general case is

$$\text{Re} = \frac{\rho_{\text{ref}} V_{\text{ref}} l_{\text{ref}}}{\mu_{\text{ref}}} \quad (2)$$

where the reference quantities in a given region are constant values representing actual significant flow quantities in that region.

It is well known that, if the Reynolds number is large, the right side of equation (1) is negligible, and the flow is said to be inviscid, except in very thin regions where the viscous terms, which contain higher order derivatives of velocities than the convective terms, are large. In the limit as  $\text{Re} \rightarrow \infty$ , it is only in regions for which the thickness is of the order of some power of  $(\text{Re})^{-1}$  that these viscous terms are important. In the inviscid limit the thin regions are represented as discontinuities in the flow variables that are changing rapidly there (the discontinuities being either at a boundary or at an internal location, where either a boundary condition or internal continuity of the solution cannot be satisfied because the order of the differential equation, or system of equations, is reduced). The structure of the flow inside these "quick-transition regions" (cf. refs. 11-13) could be analyzed by making boundary-layer-type transformations on the equations and making a uniformly valid approximate calculation by dropping out the terms of higher order in the small parameter  $(\text{Re})^{-1}$ .

Consider first the case of no mass transfer ( $V_b = 0$ ) and refer to figure 1(a), which represents schematically the flow in the forward region of a blunt body that is axially symmetric with respect to the free-stream direction. A significant parameter determining both the thickness of the shock wave,  $\delta_{sw}$ , and the boundary-layer thickness,  $\delta_{bl}$ , in comparison to the thickness of the shock layer,  $d_o$ , is

$$Re_2 = \frac{\rho_2 V_2 d_o}{\mu_2} \quad (3)$$

where subscript 2 indicates conditions on the axis immediately behind the shock wave and where  $\rho$ ,  $\mu$ , and  $V$  are, respectively, the density, viscosity, and velocity there. As is well known,

$$\left. \begin{aligned} \epsilon_s \equiv \frac{\delta_{sw}}{d_o} &= O(Re_2^{-1}) , & (V_2 - V_\infty \neq 0) \\ \epsilon_{bl} \equiv \frac{\delta_{bl}}{d_o} &= O(Re_2^{-1/2}) , & \left( \frac{d_o}{r_b} \neq 0 \right) \end{aligned} \right\} \text{ as } Re_2 \rightarrow \infty \quad (4)$$

in general. Thus, when  $V_b = 0$  and  $Re_2$  is large, the flow of air in front of the body (fig. 1(a)) is inviscid except for the thin quick-transition regions of the shock wave and the boundary layer (shaded areas).

Now consider the cases where mass transfer from the body exists ( $V_b \neq 0$ ), and in particular the regimes of flow for which  $Re_2 \gg 1$  and for which

$$Re_b \equiv \left( \frac{\rho_b V_b d_i}{\mu_b} \right)_o \quad (5)$$

is very small, of order unity, or very large (see fig. 1(b), (c), (d)). The case of small mass transfer ( $\rho_b V_b$  small) is denoted by  $Re_b \ll 1$ . The "intermediate regime" is  $Re_b = O(1)$ ; and the case of large mass transfer is denoted by  $Re_b \gg 1$ .

From the above discussion we see that  $Re_b$  is the determining factor as to whether or not the bulk of the flow from the body (i.e., the inner gas layer) is free of viscous effects. (Note appendix B.) For very small mass transfer ( $Re_b \ll 1$ ) the inner gas layer is entirely viscous and is simply part of the boundary layer (fig. 1(b)). The mass transfer does not change the order of magnitude of the boundary-layer thickness if  $Re_b \leq O(\sqrt{Re_2})$ . As the mass flux increases, say to where  $Re_b = O(1)$ , the inner layer becomes a thick viscous layer (fig. 1(c)). Mathematically speaking, it is not a boundary layer because it is not vanishingly thin as  $Re_2 \rightarrow \infty$  and the requirements for the validity of boundary-layer theory are violated, although the viscous part of the air in the outer layer does become vanishingly thin as  $Re_2 \rightarrow \infty$ . As the mass flux increases still further, so that  $Re_b \gg 1$ , the inner gas layer must become essentially inviscid (fig. 1(d)), leaving viscous effects confined to the interface layer and the boundary shock wave at the body

surface. The interface layer is essentially a "blown-off" boundary layer that has carried with it the viscous-shearing effects (effects of  $\tau_{r\phi}$ ) that characterized the boundary layer (fig. 1(b)). The viscous-compressive effects (effects of  $\tau_{rr}$ ) that became of the same order as the viscous shearing effects in the thick viscous layer (fig. 1(c)) are left confined to the boundary shock wave (fig. 1(d)). Note that no part of the flow in the boundary shock wave need be supersonic if there is heat transfer in the gas at the boundary (ref. 9). Furthermore, the boundary shock wave behaves in the same manner with varying  $Re_b$  as does a shock wave with varying  $Re_2$  (see ref. 9). Its thickness for large  $Re_b$  is of order  $\tilde{\mu}_b/\rho_b V_b$ ; that is,

$$\epsilon_{bsw} \equiv \frac{\delta_{bsw}}{d_i} = O(Re_b^{-1}) \quad \left. \begin{array}{l} \text{as } Re_b \rightarrow \infty \\ \text{with } V_e - V_b \neq 0 \end{array} \right\} \quad (6)$$

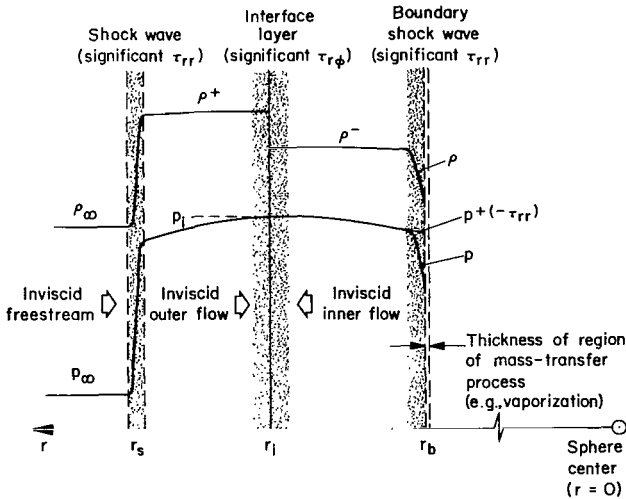
where the viscosity coefficient  $\tilde{\mu}$  is of the same order as the shear-viscosity coefficient  $\mu$ . The viscosity coefficients are related by

$$\tilde{\mu} = \frac{4}{3} \mu + \frac{1}{3} \kappa \quad (7)$$

where  $\kappa$  is the bulk viscosity coefficient (see ref. 14, p. 337). One would expect the order of magnitude of the relative thickness of the interface layer,  $\epsilon_i$ , to be given by

$$\epsilon_i \equiv \frac{\delta_i l}{d} = O(Re_2^{-1/2}) + O(Re_b^{-1/2}) \quad \text{as } Re_2 \rightarrow \infty \text{ and } Re_b \rightarrow \infty \quad (8)$$

if it can be treated like a boundary layer. However, this should probably not be assumed a priori without a careful analysis. It would be true if the tangential velocity at the interface is discontinuous in the inviscid solution.



Sketch (a)

With the assumption that the entire outer flow is governed by  $Re_2$  and the inner flow by  $Re_b$ , the limiting case for which both  $Re_2$  and  $Re_b \rightarrow \infty$  will leave two inviscid regions with possible discontinuities at their boundaries: the shock wave, the interface, and the body. Whether or not discontinuities will exist there in the inviscid limit will depend on satisfaction of the conservation equations across these vanishingly thin regions, to be considered below.

Sketch (a) may aid the understanding of flow containing a boundary shock wave. Thermodynamic pressure,  $p$ , and mass density,  $\rho$ ,



are plotted qualitatively across the several regions of flow on the axial streamline when  $Re_2$  and  $Re_b$  are both large and when some source of significant translational nonequilibrium is present at  $r_b$ . Note that in measurements of pressure, there would be no way of physically separating the thermodynamic pressure,  $p$ , from the normal viscous stress in the radial direction,  $\tau_{rr}$ . For small enough  $\rho V_b^2$ , the total normal stress in the radial direction,  $p + (-\tau_{rr})$ , does not vary significantly across the boundary shock, although the density,  $\rho$ , does if a strong boundary shock is present (i.e., with significant  $\tau_{rr}$ ).

## ASSUMPTIONS AND APPROXIMATIONS FOR FLOW CALCULATION

For the flow calculation it is assumed, first of all, that  $Re_2$  and  $Re_b$  are large enough that both the air flow in the outer layer and the flow originating from the body are essentially inviscid. It is assumed therefore that all viscous effects are confined to very thin layers at the shock wave, the interface, and at the body surface, as described above. These very thin quick-transition regions are represented by surfaces of discontinuity in the inviscid analysis. The discontinuous quantities are those that vary rapidly inside the thin viscous regions, except that the density at the interface is generally discontinuous also in the viscous flow, if diffusion is neglected. It is assumed that either the interface layer is stable near the axis of symmetry or the effects of instability are negligible (see appendix B).

Only the sphere will be considered for the body shape. In order to obtain a separable solution to the equations, the shock wave and the interface will be assumed to be concentric with the body, and the normal velocity of mass flow from the body (just outside the boundary shock wave) will vary as the cosine of the body angle  $\varphi$ . These assumptions are automatically included if we simply assume: (1) the body has constant curvature, and (2) the flow solution is separable. That is, the latter assumption is equivalent to specifying all the boundary conditions in such a way as to be compatible with a separable solution to the equations.

For simplicity, the density is taken to be constant in each of the inviscid regions, but it may vary across the surfaces of discontinuity representing rapid transitions. (The density varies rapidly across the boundary shock, as well as across the shock wave.) The outer inviscid solution will then be identical to Lighthill's constant-density solution for the sphere (ref. 3) (with the interface as the equivalent body in that solution), since the conditions across the shock wave are the same. The solution for the flow between the interface and the body will be different from those of Vinokur and Sanders (ref. 2) and Cresci and Libby (ref. 5, appendix), for the same boundary parameters, because of the presence of the boundary shock wave. (For the same values of parameters outside the boundary shock wave, the inviscid solution is the same as given in the appendix of reference 5. The relationship of the inviscid solution to the boundary parameters, under conditions where the boundary-shock-wave theory applies (ref. 10), is of most concern here.) The assumptions of constant density outside the boundary shock and the separable flow solution simply require that the mass flux from the body, as well

as the normal velocity outside the boundary shock, be specified as varying as the cosine of the body angle  $\varphi$ . These conditions will be evident when the solution is formulated.

Purely radial velocity of the mass flow out of the body surface will be assumed. The flow through the boundary shock wave will then be normal to the surface. In the solution of Vinokur and Sanders (ref. 2) the mass injection was assumed to be vectored to make the flow irrotational. The vorticity, or rotationality, of the flow from the body depends simply on the distribution of the components of velocity at the surface, which can be specified independently of the flow problem, since they depend entirely on the injection (or ablation) process.

The limiting case of very small heat conduction ( $q_{cb} \rightarrow 0$ ) in a supersonic efflux of gas at the boundary, for which the boundary shock becomes a detached shock wave (ref. 9), will not be treated here. Thus we assume  $q_{cb} \neq 0$  if  $M_b > 1$ .

The mass flux from the body may be either subsonic or supersonic for a boundary shock wave to occur (see refs. 9 and 10). We do not restrict  $M_b$  except to require  $M_b \neq 0$ , since  $M_b = 0$  would be generally incompatible with the limit  $Re_b \rightarrow \infty$ .

A value of unity is assumed for the longitudinal Prandtl number,  $\tilde{Pr} = \tilde{\mu}c_p/k_c$ , that is, the Prandtl number based on the viscosity coefficient  $\tilde{\mu}$  that occurs in the purely viscous-compressive stress,  $\tau_{rr} = \tilde{\mu}(\partial v/\partial r)$ , in the boundary shock wave. With that assumption the simple relationships developed in reference 9 for the conditions across the boundary shock wave can be used. Liepmann and his co-workers have pointed out (ref. 15) that  $\tilde{Pr} \approx 1$  for most real gases.

## INVISCID CONSERVATION EQUATIONS AND THE BOUNDARY CONDITIONS FOR THE INVISCID REGIONS

### Inviscid Conservation Equations

For steady inviscid flow the equations of conservation of mass, conservation of momentum (Euler equations), and conservation of energy are:

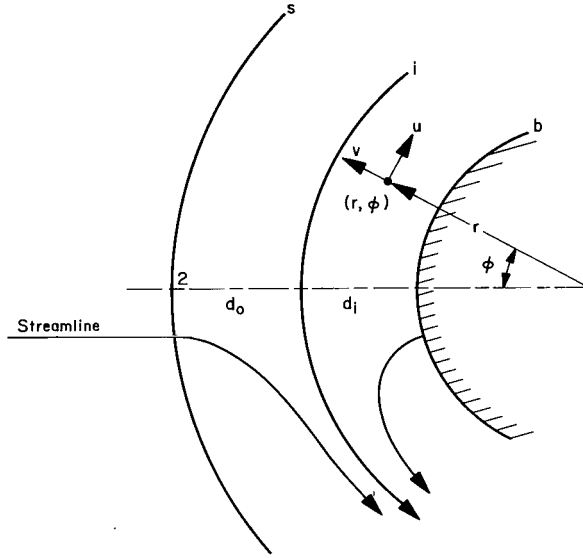
$$\text{div}(\rho \vec{V}) = 0 \quad (9)$$

$$\rho(\vec{V} \cdot \text{grad})\vec{V} + \text{grad } p = 0 \quad (10)$$

$$\rho(\vec{V} \cdot \text{grad})\left(h + \frac{1}{2} V^2\right) + \text{div } \vec{q}_r = 0 \quad (11)$$

with which appropriate equations of state and a relation for  $\vec{q}_r$  must be used. We take for the equations of state in the inviscid regions, as described above,

$$\left. \begin{aligned} \rho &= \rho^- = \text{constant}, & (r_b < r < r_b + d_i) \\ \rho &= \rho^+ = \text{constant}, & (r_b + d_i < r < r_b + d) \end{aligned} \right\} \quad (12)$$



Sketch (b)

Equations (9), (10), and (11) may be written conveniently in axisymmetric spherical coordinates  $(r, \phi)$  for the sphere (see sketch (b)).

A transformation that simplifies both the equations and their solution (but involves no approximation) is the following:

$$\zeta = \log(r/r_b) \quad (13a)$$

for which

$$r \frac{\partial}{\partial r} ( ) = \frac{\partial}{\partial \zeta} ( ) \quad (13b)$$

In terms of dimensionless quantities  $u, v, \bar{p}, \bar{\rho}$ , and  $k$ , equation (9) and the components of (10) become

$$(\bar{\rho} u e^{2\zeta} \sin \phi)_\phi + (\bar{\rho} v e^{2\zeta} \sin \phi)_\zeta = 0 \quad (14)$$

$$\bar{\rho}(u u_\phi + v u_\zeta + uv) = -k \bar{p}_\phi \quad (15a)$$

$$\bar{\rho}(u v_\phi + v v_\zeta - u^2) = -k \bar{p}_\zeta \quad (15b)$$

With dimensionless enthalpy  $\bar{h}$ , equation (11) becomes

$$\bar{\rho} u \left( \bar{h} + \frac{1}{2} u^2 + \frac{1}{2} v^2 \right)_\phi + \bar{\rho} v \left( \bar{h} + \frac{1}{2} u^2 + \frac{1}{2} v^2 \right)_\zeta = \frac{-k r \operatorname{div} \vec{q}_r}{\rho_\infty V_\infty^3} \quad (16)$$

In equations (14) to (16), the subscripts  $\phi$  and  $\zeta$  denote partial differentiation. (In these equations  $\bar{\rho}$  is still variable.) The equations of state for the inviscid regions become

$$\left. \begin{aligned} \bar{\rho} &= \bar{\rho}^- = \rho^-/\rho^+ \equiv \sigma^2 = \text{constant} & \text{for } 0 < \zeta < \zeta_i \\ \bar{\rho} &= \bar{\rho}^+ = 1 & \text{for } \zeta_i < \zeta < \zeta_s \end{aligned} \right\} \quad (17)$$

where

$$\left. \begin{aligned} \zeta_i &= \log(r_i/r_b) = \log(1 + d_i/r_b) \equiv \log D_i \\ \zeta_s &= \log(r_s/r_b) = \log(1 + d/r_b) \equiv \log D \end{aligned} \right\} \quad (18)$$

It is convenient to introduce a dimensionless Stokes stream function  $\psi$ , defined by

$$\psi_\zeta = \bar{\rho} u e^{2\zeta} \sin \varphi; \quad \psi_\varphi = -\bar{\rho} v e^{2\zeta} \sin \varphi \quad (19)$$

which automatically satisfies equation (14). Equations (15) may be combined by using equations (17) and by cross-differentiating and equating  $\bar{p}_{\zeta\varphi}$  to  $\bar{p}_{\varphi\zeta}$  to obtain a nonlinear partial differential equation for  $\psi$ , which applies for  $0 = \zeta_e < \zeta < \zeta_i$  or for  $\zeta_i < \zeta < \zeta_s$ . Because constant density is assumed for the inviscid portions of the flow, the energy equation is uncoupled from the equations for conservation of mass and momentum and, hence, need not be considered for the inviscid-flow calculation (except in the conditions across the discontinuities to determine  $\rho^-$  and  $\rho^+$ ).

#### Boundary Conditions

Since the flows on both sides of the shock wave and on both sides of the interface layer are inviscid, the inviscid-flow equations for the conservation principles can be used to determine conditions across (cf. appendix C). However, the boundary shock wave, like a boundary layer, has inviscid flow only on one side, so the conservation equations of viscous flow must be used to find conditions across the boundary shock wave. This has been done (refs. 9 and 10) for  $\widetilde{Pr} = 1$  and for a plane boundary surface. For a curved boundary surface and a very thin boundary shock wave (i.e., to first order) with flow normal to it, the results will be the same. However, the flow quantities will vary along the boundary shock wave.

The conditions behind the shock are (cf. appendix C):

at  $\zeta = \zeta_s$  :

$$v = -k \cos \varphi \quad (20a)$$

$$u = \sin \varphi \quad (20b)$$

$$\bar{p} = (1 - k) \cos^2 \varphi \quad (20c)$$

$$\bar{h} = \frac{1}{2} (1 - k^2) \cos^2 \varphi \quad (20d)$$

where  $k \equiv \rho_\infty/\rho_2$  is determined by an equation of state (or equilibrium-air tables) in combination with equations (20) (using  $\varphi = 0$ ). It will be

convenient to use (20c) in the following form (making use also of eq. (15a)):

$$\zeta = \zeta_s, \quad -k\bar{p}_\varphi = uu_\varphi + vu_\zeta + uv = +2k(1 - k)\cos \varphi \sin \varphi \quad (21)$$

The interface ( $\zeta = \zeta_i$ ) is defined as the surface between the region of gas flow that has come through the shock wave and the region of gas flow emanating from the body. This definition can be represented by

$$\zeta = \zeta_i, \quad \psi = 0 \quad (22a)$$

which, with the concentric interface (i.e.,  $\zeta_i = \text{constant}$ , not a function of  $\varphi$ ), also implies

$$\zeta = \zeta_i, \quad v = 0 \quad (22b)$$

The conditions to be applied at the inviscid interface are (cf. appendix C):

at  $\zeta = \zeta_i$ :

$$v = 0 \quad (23a)$$

$$\bar{p}^- = \bar{p}^+ \quad (23b)$$

$$\bar{\rho}^-(uu_\varphi)^- = \bar{\rho}^+(uu_\varphi)^+ \quad (23c)$$

At the body (in the inviscid flow, outside the boundary shock wave) it will be found necessary to specify:  $\bar{\rho} = \bar{\rho}_e$ ,  $v = v_e$ ,  $u = u_e$  (where  $\bar{\rho}_e = \bar{\rho}^- = \rho^-/\rho^+ \equiv \sigma^2 = \text{constant}$ ) in order that the inviscid solution be determined. (These quantities are equivalent to those "at the body" in the appendix of ref. 5.) When a boundary shock is expected, these quantities must be related to conditions at the boundary, within the viscous boundary-shock-wave region. The viscous-flow equations must be used to determine the conditions across the boundary shock, whose relative thickness (ref. 9) is  $O(\text{Re}_b^{-1})$ . For the calculation of results, one would like to specify values at the boundary, say  $M_b$ ,  $T_b$ , and the mass flux  $\rho_b V_b$ , then determine the conditions outside the boundary shock ( $\bar{\rho}_e$ ,  $v_e$ ,  $u_e$ ), which would be used as boundary conditions for the inviscid solution. However, it will be found most convenient for the calculation to specify values of a parameter  $N$  containing  $\rho_e V_{eo}^2$ . As will be seen, then,  $p_{eo}$  will be determined from the solution to the equations so that  $M_{eo}^2 = \rho_e V_{eo}^2 / \gamma p_{eo}$  will be known. If  $M_b = M_b(\varphi)$  and  $T_{bo}$  are also specified, then the complete inviscid solution, the conditions across the boundary shock, and values of quantities of interest at the boundary, including the mass flux at  $\varphi = 0$ ,  $(\rho_b V_b)_0$ , will be determined. Thus, specifying  $M_b$ ,  $T_{bo}$ , and  $\rho_e V_{eo}^2$  will give solutions corresponding to  $M_b$ ,  $T_{bo}$ , and some value of  $(\rho_b V_b)_0$ . It has been shown (refs. 9 and 10) that, for  $\text{Pr} = 1$ , perfect gas in the boundary shock, and  $q_r = \text{constant}$  across, and for known values of  $M_b$  and  $M_e$ , the conditions across the boundary shock wave for flow normal to the layer (in present nomenclature) are:

$$\left. \begin{aligned} v_e/v_b &= \alpha \\ \rho_e/\rho_b &= 1/\alpha \\ T_e/T_b &= 1 + (1/2)(\gamma - 1)M_b^2(1 - \alpha^2) = \alpha^2 M_b^2/M_e^2 \\ p_e/p_b &= \alpha M_b^2/M_e^2 \end{aligned} \right\} \quad (24)$$

where

$$\alpha \equiv \frac{M_e}{M_b} \left[ \frac{1 + (1/2)(\gamma - 1)M_b^2}{1 + (1/2)(\gamma - 1)M_e^2} \right]^{1/2} \quad (25)$$

and the heat-conduction parameter for conduction in the gas at the boundary is

$$C_{hc} \equiv \frac{-q_{cb}}{(1/2)\rho_b V_b^3} = 2 \left[ \alpha \left( 1 + \frac{1}{\gamma M_e^2} \right) - \left( 1 + \frac{1}{\gamma M_b^2} \right) \right] \quad (26)$$

To these conditions we must add for normal injection,<sup>1</sup>

$$u_e = u_b = 0 \quad (27)$$

In general, the quantities found from the above conditions are variable over  $\varphi$ . The respective variations will be specified as required to produce a separable solution for the inviscid flow, as discussed above. For a separable solution it is necessary that  $v_e$  vary as  $\cos \varphi$  and  $u_e$  at most as  $\sin \varphi$ :

at  $\zeta = \zeta_e = 0$  :

$$\left. \begin{aligned} v &= v_e = v_{eo} \cos \varphi = \frac{V_{eo}}{V_\infty} \cos \varphi \\ u &= u_e = u_{eo} \sin \varphi = 0 \end{aligned} \right\} \quad (28)$$

Once the ratios in equations (24) are known for all  $\varphi$  (with specification of  $M_b$ ,  $T_{bo}$ , and  $\rho_e V_{eo}^2$  and with  $p_e$  found from the inviscid solution), the following quantities can be calculated in order:

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<sup>1</sup>In a private communication, Prof. N. Rott (Univ. of Calif. at Los Angeles) has pointed out that if a significant tangential velocity component is present in the boundary shock wave, its treatment would be analogous to the "shock slip" condition for the tangential velocity component in an oblique shock wave (e.g., see ref. 16). This condition would have to be considered if the mass injection is "vectored" as suggested in reference 2.

$$\left.
\begin{aligned}
p_b(\varphi) &= \frac{p_e(\varphi)}{p_e/p_b} \\
T_b(\varphi) &= T_{bo} \left( \frac{T_b}{T_e} \right) \left( \frac{T_{eo}}{T_{bo}} \right) \left( \frac{p_e}{p_{eo}} \right) \left( \frac{\rho_{eo}}{\rho_e} \right) \\
&= T_{bo} \frac{(T_e/T_b)_o}{T_e/T_b} \left( \frac{p_e}{p_{eo}} \right) \\
\rho_b(\varphi) &= \frac{p_b(\varphi)}{RT_b(\varphi)} \\
\rho_e &= \rho_{eo} = \left( \frac{\rho_e}{\rho_b} \right)_o \rho_{bo} \\
V_{eo} &= \left( \frac{\rho_e V_{eo}^2}{\rho_e} \right)^{1/2} \\
\rho_b V_b &= \rho_e V_e = \rho_e V_{eo} \cos \varphi
\end{aligned}
\right\} \quad (29)$$

Different values of  $\rho_e V_{eo}^2$  could be specified initially and the solution iterated until the desired mass flux,  $\rho_b V_b$ , is obtained.

It should be noted that when the mass transfer is a result of ablation of the body surface caused by high radiative heating from the hot layer of air between the shock wave and the interface, the specification of arbitrary values of  $M_b$ ,  $T_b$ , and  $\rho_b V_b$  could be replaced by three appropriate equations similar to those used in references 9 and 10 for the one-dimensional case: a vaporization-rate equation for  $M_b$ , Clapeyron's equation (or "modified Clapeyron's eq.") for  $T_b = T_b(p_b)$ , and an equation relating  $\rho_b V_b$  to the radiative heat flux. Then the radiation is assumed to be absorbed within an infinitely thin layer just inside the body surface; some of that heat is conducted into the body, so the heat conduction into the body from the surface is not the same as the conduction in the vapor just outside the surface.

## SOLUTION OF THE EQUATIONS FOR THE INVISCID REGIONS

### Separation of Variables

The partial differential equation for  $\psi$  obtained from equations (15), (17), and (19) with the boundary conditions (20), (21), (23), and (28) has the separable solution

$$\psi = \sin^2 \varphi f(\zeta) \quad (30)$$

where the differential equation for  $f(\zeta)$  is

$$f''' - 5f'' + 2f' + 8f = 0, \quad (0 = \zeta_e < \zeta < \zeta_i \text{ or } \zeta_i < \zeta < \zeta_s) \quad (31)$$

where  $( )' \equiv d( )/d\zeta$ . The boundary conditions become:

at  $\zeta = \zeta_s = \log D$ :

$$\left. \begin{aligned} f &= kD^2/2 \\ f' &= D^2 \\ f'' &= (D^2/k)(1 - k + 2k^2) \end{aligned} \right\} \quad (32)$$

at  $\zeta = \zeta_i^+ = \log D_i$ :

$$f = 0 \quad (33)$$

at  $\zeta = \zeta_i^- = \log D_i$ :

$$\left. \begin{aligned} f &= 0 \\ (f')^2 &= [(f')^2]^- = \left(\frac{\rho^-}{\rho^+}\right) [(f')^2]^+ \end{aligned} \right\} \quad (34)$$

at  $\zeta = \zeta_e = 0$ :

$$\left. \begin{aligned} f &= \frac{-\bar{\rho}^- v_{eo}}{2} = \frac{-\rho^-}{\rho^+} \left(\frac{v_{eo}}{2}\right) \equiv \frac{-\sigma^2 v_{eo}}{2} \\ f' &= \bar{\rho}^- u_{eo} = 0 \end{aligned} \right\} \quad (35)$$

Equation (31) is a third-order linear ordinary differential equation (with constant coefficients because of the transformation (13)); its solution will have three unknown constants of integration for the region  $\zeta_i < \zeta < \zeta_s$  and three unknown constants of integration for the region  $0 < \zeta < \zeta_i$ . In addition,  $\zeta_i$  and  $\zeta_s$  (or  $D_i$  and  $D$ ) are unknown, making a total of eight unknown constants to be evaluated by the eight boundary conditions in equations (32) through (35).

#### Equations for Pressure, Velocity, and Vorticity

The boundary conditions not yet used, (20c) and (23b), will determine the constant of integration for the pressure in each of the two regions. When equations (19) and (30) are used, equations (15) become



$$-k\bar{\rho}\bar{p}_\varphi = (\sin \varphi \cos \varphi)e^{-4\zeta}[(f')^2 - 2ff'' + 2ff'] \quad (36a)$$

$$-k\bar{\rho}\bar{p}_\zeta = (\sin^2 \varphi)e^{-4\zeta}[2ff' - (f')^2] + (\cos^2 \varphi)e^{-4\zeta}[4ff' - 8f^2] \quad (36b)$$

which may be integrated to obtain

$$-k\bar{\rho}\bar{p} = (1/2)(\sin^2 \varphi)e^{-4\zeta}[(f')^2 - 2ff'' + 2ff'] + e^{-4\zeta}(2f^2) + C \quad (37)$$

where the conditions (20c) and (23b) now determine that

$$\left. \begin{aligned} C &= C^+ = \frac{1}{2} k^2 - k & \text{for } \zeta_1 < \zeta < \zeta_s \\ &= C^- = \bar{\rho}C^+ = \sigma^2\left(\frac{1}{2} k^2 - k\right) & \text{for } 0 < \zeta < \zeta_1 \end{aligned} \right\} \quad (38)$$

Outside the boundary shock wave, conditions (35) in (37) give (for  $u_{e0} = 0$ ):

$$\bar{p}_e = 1 - \frac{1}{2} k - \frac{(\sigma v_{e0})^2}{2k} - \frac{\sigma v_{e0}}{2k} \frac{f''(0)}{\sigma} \sin^2 \varphi \quad (39)$$

In terms of the function  $f(\zeta)$ , the dimensionless velocity components are:

$$\left. \begin{aligned} u &= \frac{\sin \varphi f'(\zeta)}{\bar{\rho}e^{2\zeta}} \\ v &= \frac{-2 \cos \varphi f(\zeta)}{\bar{\rho}e^{2\zeta}} \end{aligned} \right\} \quad (40)$$

and the dimensionless vorticity is

$$\left. \begin{aligned} \omega &= \frac{r_b \text{curl } \vec{V}}{V_\infty} = e^{-\zeta}(v_\varphi - u - u_\zeta) \\ &= \frac{\sin \varphi}{\bar{\rho}e^{3\zeta}} (2f + f' - f'') \end{aligned} \right\} \quad (41)$$

Integration of the Differential Equation  
and Calculation of Results

The solution to equation (31) may be written

$$\left. \begin{aligned} f(\zeta) &= f^+(\zeta) = c_1^+ e^{4(\zeta - \zeta_1)} + c_2^+ e^{2(\zeta - \zeta_1)} + c_3^+ e^{-(\zeta - \zeta_1)} \\ &= c_1^+ (r/r_1)^4 + c_2^+ (r/r_1)^2 + c_3^+ (r/r_1)^{-1} \\ &\quad (\zeta_1 < \zeta < \zeta_s) \end{aligned} \right\} \quad (42a)$$

and

$$\left. \begin{aligned} f(\zeta) &= f^-(\zeta) = c_1^- e^{4(\zeta - \zeta_1)} + c_2^- e^{2(\zeta - \zeta_1)} + c_3^- e^{-(\zeta - \zeta_1)} \\ &= c_1^- (r/r_1)^4 + c_2^- (r/r_1)^2 + c_3^- (r/r_1)^{-1} \\ &\quad (0 < \zeta < \zeta_1) \end{aligned} \right\} \quad (42b)$$

The constants  $c_1^+$ ,  $c_2^+$ ,  $c_3^+$ ,  $c_1^-$ ,  $c_2^-$ ,  $c_3^-$ ,  $D$ , and  $D_1$  are evaluated by substituting conditions (32) through (35) into the solution (42). The results are

$$\frac{c_1^+}{D_1^2} = \frac{(1 - k)^2}{10k\bar{D}^2}; \quad \frac{c_2^+}{D_1^2} = \frac{4k - 1}{6k}; \quad \frac{c_3^+}{D_1^2} = \frac{(1 - k)(1 - 6k)\bar{D}^3}{15k} \quad (43)$$

where

$$\bar{D} \equiv \frac{D}{D_1} = \frac{r_b + d}{r_b + d_1} = \frac{r_s}{r_1} \quad (44)$$

is the appropriate root of

$$3(1 - k)^2 + 5(4k - 1)\bar{D}^2 + 2(1 - k)(1 - 6k)\bar{D}^5 = 0 \quad (45)$$

Also obtained are:

$$\left. \begin{aligned} \frac{c_1^-}{\beta\sigma} &= \left( D_1^5 + 2D_1^2 - 3 \frac{\sigma u_{eo}}{\beta} D_1^2 \right) \frac{D_1^2}{2B} \\ \frac{c_2^-}{\beta\sigma} &= \left( -D_1^5 - 4 + \frac{\sigma u_{eo}}{\beta} D_1^2 \right) \frac{D_1^2}{2B} \\ \frac{c_3^-}{\beta\sigma} &= \left( -2D_1^2 + 4 + 2 \frac{\sigma u_{eo}}{\beta} D_1^2 \right) \frac{D_1^2}{2B} \end{aligned} \right\} \quad (46)$$

where  $D_i$  is the appropriate root of

$$N - \frac{A(D_i)}{B(D_i)} = 0 \quad (47)$$

and where

$$N \equiv \frac{\sigma v_{e0}}{\beta} = \sqrt{\frac{k}{\beta^2} \frac{\rho_e v_{e0}^2}{\rho_\infty v_\infty^2}} \quad (48)$$

$$\sigma^2 \equiv \rho^-/\rho^+ = \bar{\rho}^- = \bar{\rho}_e = k\rho_e/\rho_\infty \quad (49)$$

$$\left. \begin{aligned} \beta = \beta(k) &\equiv \frac{u(\zeta = \zeta_i^+)}{\sin \varphi} = \frac{\sigma u(\zeta = \zeta_i^-)}{\sin \varphi} \\ &= 4\left(\frac{c_1^+}{D_i^2}\right) + 2\left(\frac{c_2^+}{D_i^2}\right) - \left(\frac{c_3^+}{D_i^2}\right) \\ &= \frac{1}{15k} \left[ \frac{6(1-k)^2}{\bar{D}^2} + 5(4k-1) - \bar{D}^3(1-k)(1-6k) \right] \end{aligned} \right\} \quad (50)$$

and

$$A(D_i) = [3 - 2(\sigma u_{e0}/\beta)] D_i^5 - 5D_i^3 - (\sigma u_{e0}/\beta) D_i^2 + 2 + 3(\sigma u_{e0}/\beta) \quad (51)$$

$$B(D_i) = D_i^5 + 5D_i^2 - 6 \quad (52)$$

For small  $d_i/r_b$  ( $D_i \approx 1$ ), equation (47) may be expanded (for  $u_{e0} = 0$ ) as:

$$N = \frac{d_i}{r_b} + \frac{2}{3} \left(\frac{d_i}{r_b}\right)^2 - \frac{1}{3} \left(\frac{d_i}{r_b}\right)^3 + \dots \quad (53)$$

and the inverse expansion, conveniently used for evaluating  $d_i/r_b$  for given small  $N$ , is

$$\frac{d_i}{r_b} = N - \frac{2}{3} N^2 + \frac{11}{9} N^3 - \dots \quad (u_{e0} = 0) \quad (54)$$

It is particularly useful to note that  $f^+(\zeta)/D_i^2$  depends only on  $k$  (since  $c_1^+/D_i^2$ ,  $c_2^+/D_i^2$ , and  $c_3^+/D_i^2$  are functions of  $k$  and  $\bar{D}$ , and  $\bar{D} = \bar{D}(k)$ ), so that  $(1/D_i^2)f^+$  and its derivatives can be calculated versus  $\zeta$  independently of the solution for  $0 < \zeta < \zeta_i$ . Similarly,  $f^-(\zeta)/\beta\sigma$  depends

only on the parameter  $N = \sigma v_{eo}/\beta$  for  $u_{eo} = 0$  (since  $c_1^-/\beta\sigma$ ,  $c_2^-/\beta\sigma$ , and  $c_3^-/\beta\sigma$  are functions only of  $D_1$  for  $u_{eo} = 0$ , and  $D_1 = D_1(N)$  for  $u_{eo} = 0$ ), and can be computed independently of the solution for  $\xi_1 < \xi < \xi_s$ .

Another result of special interest here is that, for a given flight condition (given  $V_\infty$ ,  $\rho_\infty$ ,  $p_\infty$ ), for which  $k$  and  $\beta$  are known,  $f''(\xi = 0)/\sigma$  is known so that  $\bar{p}_e$  (eq. (39)) depends only on  $\sigma v_{eo}$  or on  $N$  (eq. (48)); hence,  $\bar{p}_e$  can be found for given  $N$  regardless of the value of either  $\sigma$  or  $v_{eo}$ . Then the conditions across the boundary shock wave can be worked out in the manner outlined in the section "Boundary Conditions," by specifying values of  $M_b$ ,  $T_{bo}$ , and  $\rho_e v_{eo}^2$  or  $N$ . (See eqs. (24) through (29).) Equivalently (and often more conveniently) one could specify different values of  $D_1$  (rather than  $N$ ), for which  $N$  is easily calculated from equation (47).

## DISCUSSION OF RESULTS

The ratio of the width of the outer inviscid region to the radius of the interface, shown in figure 2(a), is identical to that given by Vinokur and Sanders (ref. 2) and by Cresci and Libby (ref. 5); values of that ratio are also identical to the values given by Lighthill's solution (ref. 3) for the ratio of the shock standoff distance to the body radius for the same values of  $k \equiv \rho_\infty/\rho_2$  in the case of no mass injection. Shown in figure 2(b) is the ratio of the thickness of the inner inviscid region to the body radius versus the parameter  $N \equiv \sigma v_{eo}/\beta$ . Note that this "inviscid blowing parameter"  $N$  is based on conditions outside the boundary shock wave, and not on conditions immediately adjacent to the body surface when a boundary shock is present.

The dimensionless tangential velocity on either side of the interface ( $u_1^+$  or  $u_1^-$ ) can be found from the function  $\beta$  plotted versus  $k \equiv \rho_\infty/\rho_2$  in figure 3. This function  $\beta$  is identical to the dimensionless stagnation-point velocity gradient from Lighthill's solution.

Figure 4 shows examples of the tangential-velocity profile for the particular cases for which  $k = 0.06$  and  $d_i/r_b = 0.3$ . Note that the function plotted in figure 4(a) is continuous at the interface, but that the velocity itself is discontinuous if  $\rho^-/\rho^+$  is not unity (see fig. 4(b), in which  $\rho^-/\rho^+ = 2$ ).

The remaining calculations, for quantities at or near the body surface (figs. 5 to 11), were all made for  $V_\infty = 20.83$  km/sec, altitude = 60 km (for which  $k = 0.06554$  and  $\beta = 0.4190$ ). The ratio of specific heats in the gas emanating from the body was taken to be  $\gamma = 7/5$ . To illustrate the variation of quantities along the boundary shock wave (i.e., with varying  $\phi$ ) in figures 5, 6, and 7, the particular value 0.025 was arbitrarily chosen for the parameter  $N$ . Figure 5 shows the variation of  $p_e$ ,  $\rho_e v_e^2$ , and  $M_e$ . Note that these variations correspond to the assumed cosine distribution for the velocity outside the boundary shock wave. For figures 6 and 7 it was also necessary to choose a value for  $M_b$ . For these calculations,  $M_b = 1/3$  was arbitrarily chosen. The values for  $T_{bo}$  and the molecular weight  $m$  need not be specified for these results.

For the same flight condition ( $V_\infty = 20.83$  km/sec, altitude = 60 km), the values of various quantities of interest relating to the boundary shock wave in the gas on the axis at or very near the body surface are shown plotted versus the inviscid blowing parameter  $N$  (for  $\gamma = 7/5$  in the inner region) in figures 8 through 11. One need not specify either  $M_b$ ,  $T_{bo}$ , or  $m$  in figure 8, but calculating the ratios across the boundary shock, conditions at the body surface, and certain conditions outside the boundary shock wave requires that  $M_b$  be specified (figs. 9 to 11). It is pertinent to note again (as in the discussion of sketch (a)) regarding the pressure variation across the boundary shock, that a measured pressure normal to the  $r$  direction would be the total normal stress,  $p + (-\tau_{rr})$ , rather than the purely thermodynamic pressure  $p$ . Note also that the heat conduction parameter  $C_{hc}$  (fig. 10(a)) expresses the heat conduction in the gas at the boundary; its relationship to the heat conduction inside the body surface depends on the mechanism of the mass transfer (refer to refs. 9 and 10). For all the quantities plotted in these figures, neither  $T_{bo}$  nor  $m$  need be specified. Worthy of particular notice is the result shown in figure 10(d), the relationship between the mass flux  $(\rho_b V_b)_0$  and the parameter  $N$  referred to in above discussion.

If one makes the approximation

$$\bar{p}_e \equiv \frac{p_e - p_\infty}{\rho_\infty V_\infty^2} \approx \frac{p_e}{\rho_\infty V_\infty^2} \quad (55)$$

one can plot the results given in figures 5 through 11 in more general form. Then, the solution for appropriate dimensionless quantities would depend only on  $k$ , rather than on the entire flight condition with specified altitude and flight velocity. For example, with the flight condition used in figure 5, for which  $\rho_\infty V_\infty^2 = 1.533 \times 10^6$  dynes/cm<sup>2</sup>,

$$\left. \begin{aligned} \frac{\rho_e V_e^2}{10^3 \text{ dynes/cm}^2} &= 1533 \left( \frac{\rho_e V_e^2}{\rho_\infty V_\infty^2} \right) \\ \text{and} \quad \frac{p_e}{10^6 \text{ dynes/cm}^2} &= 1.533 \left( \frac{p_e}{\rho_\infty V_\infty^2} \right) \end{aligned} \right\} \quad (56)$$

so that, with the approximation (55), the values on the three curves on figure 5 are equivalent, respectively, to the two quantities on the right sides of equations (56) and  $M_e$  for just  $k = 0.06554$ ,  $N = 0.025$ , and  $\gamma = 7/5$ . Similarly (with use of the approximation (55) and with the flight condition (altitude and velocity) replaced by  $k = 0.06554$ ), the results of figure 6 apply directly, and those on figure 7 for  $C_{hc}/100$  and  $T_b/T_{bo}$  apply directly,

whereas the values on the curve labeled  $\left( \frac{50 \text{ gm/mole}}{m} \right) \left( \frac{T_{bo}}{1^\circ \text{K}} \right) \left( \frac{\rho_b}{1 \text{ gm/cm}^3} \right)$  become

the values of  $\left(\frac{50 \text{ gm/mole}}{m}\right) \left(\frac{T_{bo}}{10^4 K}\right) \left(\frac{1.533 \times 10^6 \text{ cm}^2/\text{sec}^2}{V_\infty^2}\right) \left(\frac{\rho_b}{\rho_\infty}\right)$ . The remaining results of figures 7 to 11 can be worked out for the more general condition in a similar manner (with, e.g., the quantities  $M_e$ ,  $p_e/p_b$ ,  $\rho_e/\rho_b$ ,  $T_e/T_b$ ,  $Ch_c$  applying directly). Of special interest is a further result, plotted in figure 12: the dimensionless heat-conduction flux in the gas at the body versus another "blowing parameter," based on conditions at the boundary,

$$F \equiv \left(\frac{\rho_{bo} V_{bo}^2}{\rho_\infty V_\infty^2}\right)^{1/2} \quad (57)$$

showing values on various lines of constant  $N$  or lines of constant  $M_b$ . For given  $N$  and  $M_b$ , the blowing parameter is calculated from

$$\frac{\rho_b V_{bo}^2}{\rho_\infty V_\infty^2} = \left(\frac{\rho_e}{\rho_b}\right)_0 \frac{\rho_e V_{eo}^2}{\rho_\infty V_\infty^2} = \left(\frac{\rho_e}{\rho_b}\right)_0 \frac{\beta^2 N^2}{k} \quad (58)$$

and the dimensionless heat conduction is found in the form

$$\left(\frac{V_\infty^2}{RT_\infty}\right)^{1/2} \left(\frac{-q_{cb}}{\frac{1}{2} \rho_\infty V_\infty^3}\right)_0 = \gamma^{1/2} M_{bo} F^2 (Ch_c)_0 \quad (59)$$

Obviously, the variation of the heat transfer with the blowing parameter depends on the simultaneous variation of  $M_b$ . For example, if  $M_b$  remained constant as  $F$  increased (where  $M_b = \sqrt{\rho_\infty V_\infty^2} (F/\gamma p_{bo})$ ), then

$(V_\infty^2/RT_{bo})^{1/2} [-q_{cb}/(1/2) \rho_\infty V_\infty^3]$  would decrease. As pointed out above (paragraph following eqs. (29)), the complete determination depends on other conditions of the problem, including the mechanism of the mass transfer.

In discussing the above results it must be recognized that the boundary-shock-wave theory, and the inviscid-flow solution, are only valid asymptotically as  $Re_b \rightarrow \infty$ . Hence, for sufficiently small  $N$  or  $F$ , the results given here are not valid. As  $N$  and  $F$  approach zero, the inner flow becomes fully viscous and then approaches a boundary-layer-type flow, so that, in the limit, the ratio  $p_e/p_b$  must go to unity; the heat conduction becomes the usual convective heat transfer of boundary-layer theory; the normal viscous stress becomes insignificant in comparison to the thermodynamic pressure  $p$ ; and the viscous shear,  $\tau_{r\phi}$ , becomes significant. (However, even a value of  $N$  equal to 0.02 or 0.01 can correspond to a substantially high value of  $Re_b$ .) As an example, if for sufficiently large  $N$  a constant value of  $M_b$  were appropriate as  $N$  varies (e.g.,  $M_b \approx 1/3$ , as estimated in refs. 9 and 10), then as  $N \rightarrow 0$ , the variation of  $p_e/p_b$  would be expected to follow a curve such as the qualitative dashed curve sketched on figure 9(a). Similarly, the value of  $Ch_c$  in figure 10(a) would first approach a curve of variation appropriate

to "moderately large" values of mass flux, and eventually to that appropriate to boundary-layer theory as  $N$  and  $Re_D$  approach zero.

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# APPENDIX A

## PRINCIPAL NOTATION

$C$	constant of integration in pressure; evaluated in equation (38)
$C_{hc}$	heat-conduction parameter, equation (26)
$c_p, c_v$	specific heats at constant pressure and constant volume, respectively
$c_1, c_2, c_3$	constants of integration; evaluated in equations (43) and (46)
$D$	$1 + \frac{d}{r_b} = \frac{r_s}{r_b}$
$\bar{D}$	$\frac{D}{D_i}$ (eq. (44))
$D_i$	$1 + \frac{d_i}{r_b} = \frac{r_i}{r_b}$
$d$	$d_o + d_i$
$d_i$	thickness of the layer of gas emanating from the body
$d_o$	thickness of the layer of air between the shock wave and the interface
$F$	$\left( \frac{\rho_{bo} V_{bo}^2}{\rho_\infty V_\infty^2} \right)^{1/2}$
$f$	function of $\xi$ in the stream function, equation (30)
$h$	specific enthalpy
$\bar{h}$	$\frac{h - h_\infty}{V_\infty^2}$
$k$	shock density ratio, $\frac{\rho_\infty}{\rho_2}$
$k_c$	coefficient of thermal conductivity
$M$	Mach number, $\frac{V}{\text{speed of sound}}$



m	molecular weight
N	$\frac{\sigma v_{eo}}{\beta} = \left( \frac{k}{\beta^2} \frac{\rho_e V_{eo}^2}{\rho_\infty V_\infty^2} \right)^{1/2}$
$\widetilde{Pr}$	$\frac{\tilde{\mu} c_p}{k_c}$ (Prandtl number based on $\tilde{\mu}$ )
p	thermodynamic pressure
$\bar{p}$	$\frac{p - p_\infty}{\rho_\infty V_\infty^2}$
q	magnitude of heat flux
$\vec{q}$	heat flux vector
R	gas constant, $\left(\frac{1}{m}\right)$ times universal gas constant
$Re_b$	$\left( \frac{\rho_b V_b d_i}{\mu_b} \right)_o$ , Reynolds number
$Re_2$	$\frac{\rho_2 V_2 d_o}{\mu_2}$ , Reynolds number
r	distance from the sphere center
$\bar{r}$	$\frac{r}{r_i}$
T	temperature
u	dimensionless tangential velocity, $\frac{1}{V_\infty} \times$ tangential velocity
V	magnitude of velocity
$\vec{V}$	velocity vector
v	dimensionless radial velocity, $\frac{1}{V_\infty} \times$ radial velocity
Z	magnified independent variable in thin region; product of $\zeta$ and reciprocal of some small dimension, depending on the region; see appendix C
$\alpha$	$\frac{\rho_b}{\rho_e}$ (eqs. (24) and (25))

$\beta$	function of $k$ (eqs. (50)); equivalent to stagnation-point velocity gradient in Lighthill solution
$\gamma$	ratio of specific heats, $\frac{c_p}{c_v}$ , in the gas from the body
$\delta_{bl}$	boundary-layer thickness
$\delta_{bsw}$	thickness of the boundary shock wave
$\delta_{il}$	thickness of the interface layer
$\delta_{sw}$	thickness of the detached shock wave in the air
$\epsilon_{bl}$	$\frac{\delta_{bl}}{d_o}$ , equations (4)
$\epsilon_{bsw}$	$\frac{\delta_{bsw}}{d_i}$ , equation (6)
$\epsilon_i$	$\frac{\delta_{il}}{d}$ , equation (8)
$\epsilon_s$	$\frac{\delta_{sw}}{d_o}$ , equations (4)
$\zeta$	dimensionless independent variable, equation (13a)
$\kappa$	bulk viscosity coefficient
$\mu$	shear viscosity coefficient
$\tilde{\mu}$	$\left(\frac{4}{3}\right)\mu + \left(\frac{1}{3}\right)\kappa$ (eq. (7))
$\rho$	mass density
$\bar{\rho}$	$\frac{\rho}{\rho_2}$
$\sigma$	$\sqrt{\frac{\rho^-}{\rho^+}} = \sqrt{\frac{\rho_e}{\rho_s}}$
$\tau_{r\phi}, \tau_{rr}$	respectively viscous-shear stress and viscous-compressive stress in the radial direction
$\phi$	angle between a radial line through a point in the flow and the body-free-stream axis
$\psi$	dimensionless Stokes stream function, defined by equations (19)
$\omega$	dimensionless vorticity, equation (41)

### Subscripts

b	value just outside the body surface
c	heat conduction
e	value just outside the boundary shock wave
e0	value outside the boundary shock at $\varphi = 0$
i	value at the interface
o	value at $\varphi = 0$
r	radiative heat transfer (except in $\tau_{r\varphi}$ and $\tau_{rr}$ )
s	value behind the shock wave
$\zeta$	partial derivative with respect to $\zeta$
$\varphi$	partial derivative with respect to $\varphi$ (except in $\tau_{r\varphi}$ )
2	value behind the shock wave at $\varphi = 0$
$\infty$	value in the free stream

### Superscripts

+	value in the outer air layer, between the interface and the shock wave
-	value in the inner gas layer, between the boundary shock wave and the interface

## APPENDIX B

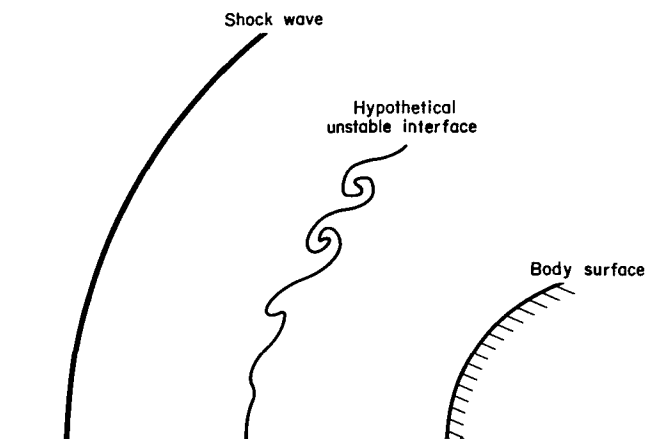
### ON THE POSSIBILITY OF STANDING EDDIES OR AN UNSTABLE INTERFACE LAYER

We may consider the possibility of regions internal to the flow whose essential character is not governed by either  $Re_2$  or  $Re_b$ . (See appendix A for notation.) The steady flow immediately behind the shock wave is certainly governed by  $Re_2$  and that adjacent to the body is certainly governed by  $Re_b$ . But suppose there is some factor at the interface not accounted for that would allow one or more regions (standing eddies) of dimension, say,  $l^*$ , to develop in which typical values of  $\rho$ ,  $V$ , and  $\mu$  are  $\rho^*$ ,  $V^*$ , and  $\mu^*$ . Then a Reynolds number

$$Re^* = \frac{\rho^* V^* l^*}{\mu^*} \quad (B1)$$

could actually be the governing parameter in that region; and if, for example,  $Re^*$  were of order unity, then there would be a region of viscous flow not of vanishing thickness as  $Re_b$  and  $Re_2 \rightarrow \infty$ . Such an event is considered to be unlikely, and is simply assumed not to occur when the flow calculation is made in the text of the report.

There is another related possibility that can be eliminated only after careful consideration: instability of the interface and the formation of eddies. The interface between the air and the gas flowing from the body is a contact surface of discontinuity. In the inviscid limit, in the general case, the tangential velocity is discontinuous (shown in appendix C) and the interface layer is a "vortex sheet." According to Rouse (ref. 17, p. 303): "Since the time of Helmholtz it has been understood that a vortex sheet is inherently unstable and will degenerate into a series of vorticity concentrations if the slightest disturbance is present." Prandtl (ref. 18, pp. 50-53) discussed several examples of surfaces of discontinuity, all of which break up into eddies. In Prandtl's words: "Owing to fluctuations in the flow the surface of separation may take on a slightly wavy form . . . . The waves advance with a velocity which is equal to the mean of the two velocities . . . ." Such a surface of discontinuity then ". . . breaks down into separate eddies" (ref. 18, pp. 50, 51). Prandtl described further the mechanism of the instability. According to Prandtl's descriptions one might expect the interface (for both  $Re_2$  and  $Re_b$  very large) to look like the purely hypothetical case depicted in sketch (c). Perhaps some physical reasoning can be added that will help to explain the development into eddies in the other examples and will also bring out a contrasting situation in the present problem. In all the examples discussed by Prandtl and in all others (at least in subsonic flow) known to the present writer, the pressure in the region where a contact surface is breaking up into eddies is always increasing with streamwise distance. When a contact surface of discontinuity is formed in the flow over a sharp edge, the breakup (instability) appears to begin at the point where one would expect



Sketch (c).- Hypothetical unstable interface.

the pressure to begin increasing (where the streamlines begin diverging<sup>1</sup>). Hence the contact surface is unstable in an "adverse pressure gradient." The contact surface of discontinuity is really a very thin viscous layer, like a boundary layer, between two flows at high Reynolds number. A boundary layer tends to separate from its boundary in the presence of an adverse pressure gradient (see discussions by Schlichting, ref. 19, pp. 23 and 26-32, and Prandtl, ref. 18, p. 137), which is always associated with the formation of vortices (ref. 19, p. 23). Since the "internal boundary layer" at the surface of "discontinuity" does not have a wall to separate from in the presence of the

adverse pressure gradient, the vorticity concentrations tend to separate from each other and so to form eddies in the same way as a separating boundary layer does. Thus it is conjectured that the same phenomenon causes the instability and collapse of a contact surface that causes separation of a boundary layer, which only occurs in the presence of an adverse pressure gradient. It is further conjectured, then, that the Helmholtz instability and eddy formation may not occur at a contact surface in the presence of a favorable pressure gradient. In the present problem, the streamlines are known to be converging and the pressure decreasing. Therefore the eddy formation and collapse of the contact surface are not expected to occur in this problem, especially near the axis of symmetry.

Since it has not been proved here that the instability will not occur, some further remarks may be made. First, it is known that the tendency toward instability and eddy formation is greater the higher the Reynolds number. Hence, the Reynolds number  $Re_p$  may be large enough that asymptotic solutions for high  $Re_p$  are useful, but  $Re_p$  may not be too large to prevent viscous damping of disturbances in the flow pattern that would initiate the eddy formation (cf. ref. 17, p. 303). That is, the viscous interface layer may be thick enough that an unstable "discontinuity configuration" of the flow is not too closely approached.

Second, even if  $Re_p$  is very large and if the instability and eddy-formation condition can develop in spite of the above conjectures, the region in which the eddies occur may be thin enough not to influence the inviscid flow significantly, but merely to influence the structure of the viscous interface layer, at least in the region of interest near the axis of symmetry.

<sup>1</sup>From conservation of mass, one finds diverging streamlines to have decreasing velocity and, hence, from Bernoulli's equation, increasing pressure.

Third, if the fluid densities in the two inviscid regions are the same, then there is no discontinuity in the inviscid velocities and hence no likelihood of an unstable interface.

## APPENDIX C

### DERIVATION OF INVISCID SHOCK-WAVE AND INTERFACE

#### BOUNDARY CONDITIONS

The main purpose of this appendix is to show a consistent and rigorous derivation of the conditions applying across the interface layer. Because of the convenience of the direct analogy to a similar derivation of conditions across the shock wave, the formal derivation of conditions across the shock is shown first.

To derive the conditions across the shock wave and the interface, consider the shock to have relative thickness  $\epsilon_s$  and the interface layer to have relative thickness  $\epsilon_i = \epsilon_i^+ + \epsilon_i^-$ . (See appendix A for notation.) These thin quick-transition regions are parallel to constant  $\xi$  lines; that is, the flow quantities vary rapidly over  $\xi$ . For the shock wave define  $Z = \xi/\epsilon_s$  so that  $\Delta Z = O(1)$  across the shock wave. Assume  $\bar{q}_r$  is constant across the shock wave (i.e., no significant radiation is absorbed or emitted within the shock). Then the Navier-Stokes equations for the flow through the shock are (cf. eqs. (14), (15), and (16)):

$$\left. \begin{aligned} (\bar{\rho}v)_Z + O(\epsilon_s) &= 0 \\ \bar{\rho}vu_Z + O(\epsilon_s) + \text{viscous terms} &= 0 \\ \bar{\rho}vv_Z + k\bar{p}_Z + O(\epsilon_s) + \text{viscous terms} &= 0 \\ \bar{\rho}v\left(\bar{h} + \frac{1}{2}u^2 + \frac{1}{2}v^2\right)_Z + O(\epsilon_s) + \text{viscous terms} &= 0 \end{aligned} \right\} \quad (C1)$$

from which we see that, in the inviscid limit ( $\epsilon_s \rightarrow 0$ ; viscous terms  $\rightarrow 0$  on both sides),

$$\left. \begin{aligned} \bar{\rho}v &= \text{constant} \\ u &= \text{constant} \\ (\bar{\rho}v)v + k\bar{p} &= \text{constant} \\ \bar{\rho}v\left(\bar{h} + \frac{1}{2}u^2 + \frac{1}{2}v^2\right) &= \text{constant} \end{aligned} \right\} \quad \text{across the shock} \quad (C2)$$

that is, the inviscid conservation equations are satisfied across the discontinuity representing the quick transition region only if equations (C2) are

satisfied. These well-known conditions across a shock wave give the conditions behind the shock as equations (20) in the text of the report.

The interface ( $\zeta = \zeta_i$ ) is defined as the surface between the region of gas flow that has come through the shock wave and the region of gas flow emanating from the body. (See eqs. (22) in the text.) The conditions across the interface can be derived in a manner analogous to that used for the shock wave. Let  $\epsilon_i^+$  be the relative thickness of the air part of the interface layer ( $\zeta > \zeta_i$ ) and  $\epsilon_i^-$  be the relative thickness of the part in  $\zeta < \zeta_i$ . Now let  $Z = (\zeta - \zeta_i)/\epsilon_i^+$  for  $\zeta > \zeta_i$  and  $Z = (\zeta - \zeta_i)/\epsilon_i^-$  for  $\zeta \leq \zeta_i$ , so that  $\Delta Z = O(1)$  across the interface layer. It can be shown that, in the analogy with the boundary layer, if  $u$  varies rapidly across the interface layer, then  $v = O(\epsilon_i) = \epsilon_i W$  inside the viscous layer, where  $\epsilon_i = \epsilon_i^+ = O(\text{Re}_2)^{-1/2}$  for  $\zeta > \zeta_i$  and where  $\epsilon_i = \epsilon_i^- = O(\text{Re}_b)^{-1/2}$  for  $\zeta < \zeta_i$ . Assuming  $\vec{q}_r$  is constant across the interface layer, we can represent the Navier-Stokes equations (cf. again eqs. (14), (15), and (16)) as:

$$(\bar{\rho} u \sin \varphi)_\varphi + (\bar{\rho} W \sin \varphi)_Z + O(\epsilon_i^-) + O(\epsilon_i^+) = 0 \quad (\text{C3a})$$

$$\bar{\rho}(uu_\varphi + Wu_Z) + k\bar{p}_\varphi + O(\epsilon_i^-) + O(\epsilon_i^+) + \text{viscous terms} = 0 \quad (\text{C3b})$$

$$k\bar{p}_Z + O(\epsilon_i^-) + O(\epsilon_i^+) + \text{viscous terms} = 0 \quad (\text{C3c})$$

$$\bar{\rho} \left[ u \left( \bar{h} + \frac{1}{2} u^2 \right)_\varphi + W \left( \bar{h} + \frac{1}{2} u^2 \right)_Z \right] + O(\epsilon_i^-) + O(\epsilon_i^+) + \text{viscous terms} = 0 \quad (\text{C3d})$$

In the inviscid limit ( $\epsilon_i^- \rightarrow 0$ ,  $\epsilon_i^+ \rightarrow 0$ , viscous terms  $\rightarrow 0$  on both sides of the interface), equations (C3a) and (C3d) give no information about conditions across the discontinuity. Equation (C3a) is automatically satisfied, since mass does not cross the interface. Equation (C3d) simply says that  $\bar{h} + (1/2)u^2$  is constant along the inviscid streamlines immediately adjacent to both sides of the interface. Equations (C3c) and (C3b) show, respectively, that the inviscid momentum equations are satisfied across the discontinuity representing the interface layer only if

$$\left. \begin{aligned} \bar{p} &= \text{constant across the inviscid interface discontinuity} \\ \bar{\rho}(uu_\varphi + v u_\zeta) + k\bar{p}_\varphi &= 0 \quad \text{on each side of the inviscid interface discontinuity} \end{aligned} \right\} \quad (\text{C4a})$$

Since  $\bar{p}$  is continuous across, from the first equation, and  $v = 0$  on both sides (eq. (22b)), the second equation requires

$$\bar{\rho} u u_\varphi = \text{continuous across the inviscid interface} \quad (\text{C4b})$$

The conditions to be applied at the inviscid interface are therefore equations (23) in the text of the report.

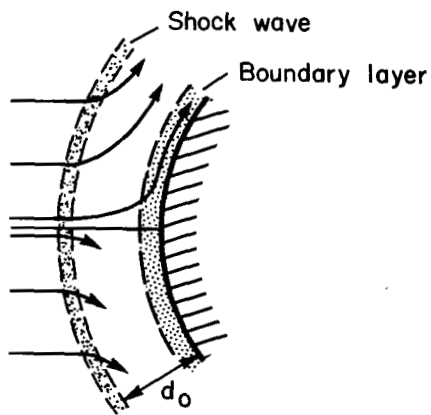


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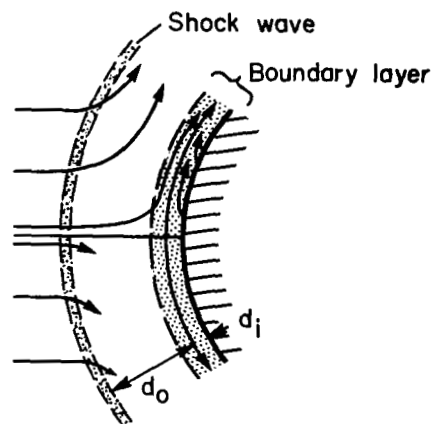
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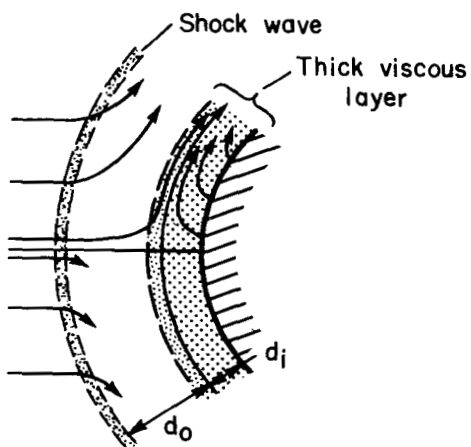




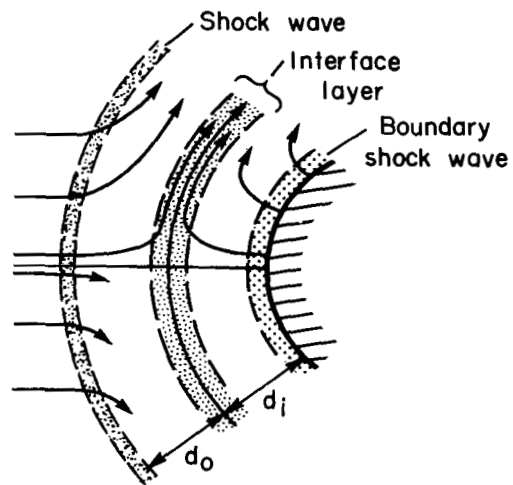
(a)  $\frac{\rho_2 V_2 d_0}{\mu_2} \gg 1; V_b = 0$



(b)  $\frac{\rho_2 V_2 d_0}{\mu_2} \gg 1; \frac{\rho_b V_b d_i}{\mu_b} \ll 1$

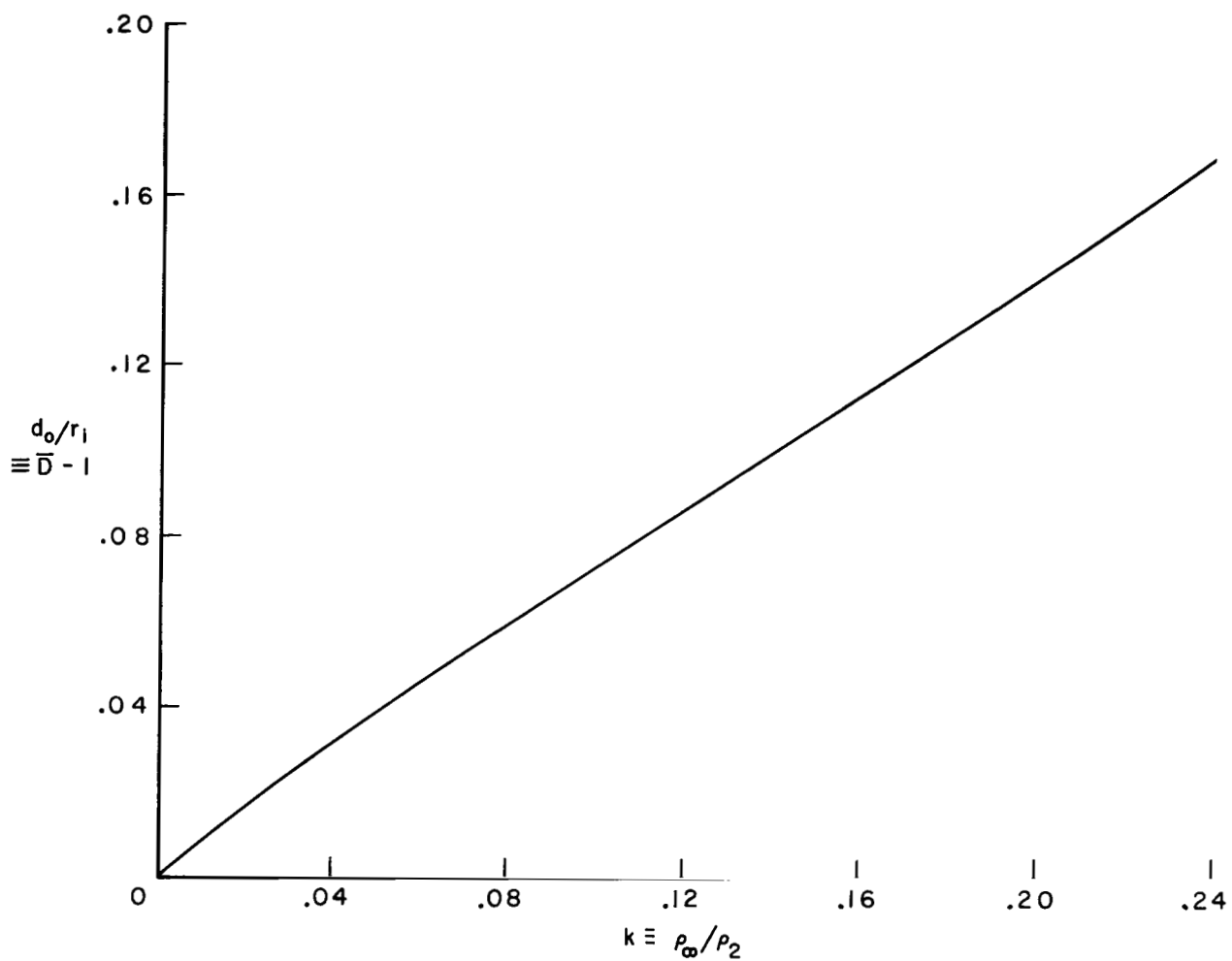


(c)  $\frac{\rho_2 V_2 d_0}{\mu_2} \gg 1; \frac{\rho_b V_b d_i}{\mu_b} = O(1)$



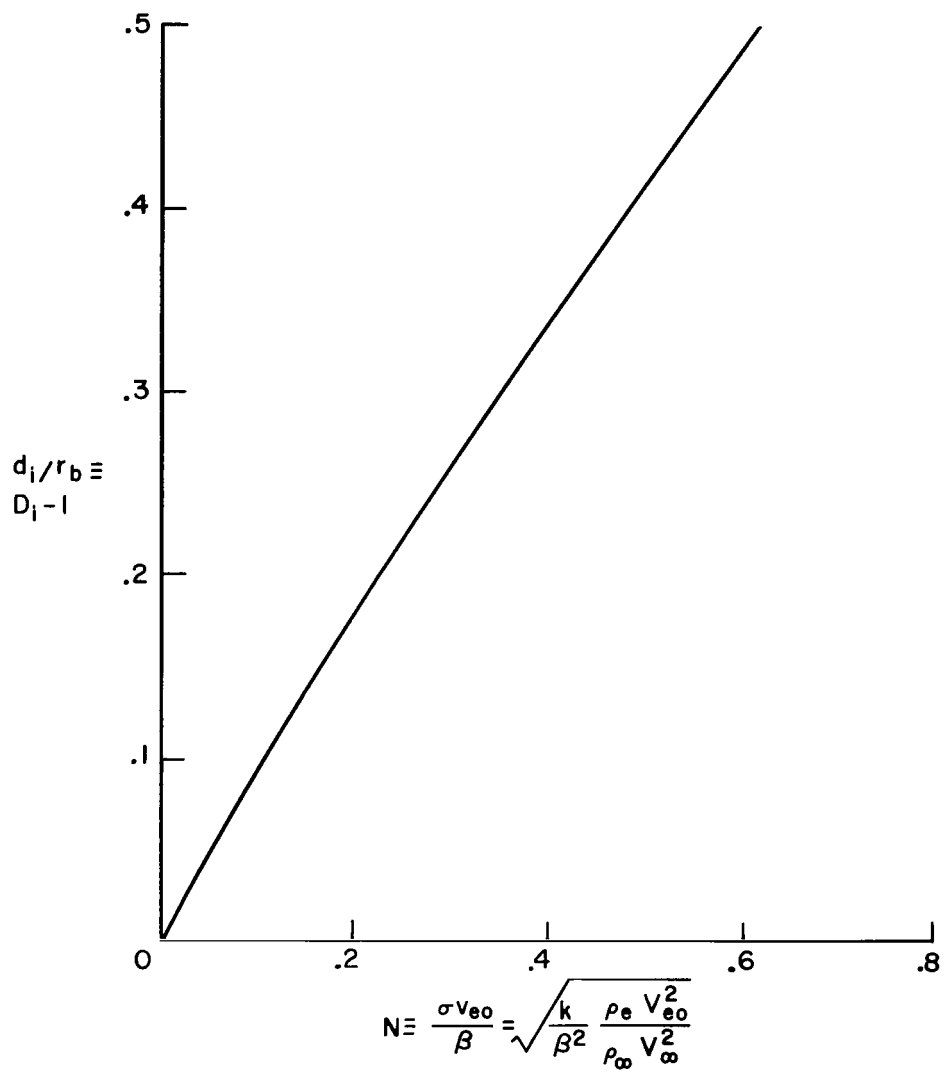
(d)  $\frac{\rho_2 V_2 d_0}{\mu_2} \gg 1; \frac{\rho_b V_b d_i}{\mu_b} \gg 1$

Figure 1.- Mass-transfer flow regimes.



(a) Thickness of outer layer,  $d_o$ .

Figure 2.- Shock wave and interface standoff functions.



(b) Thickness of inner layer,  $d_i$ .

Figure 2.- Concluded.

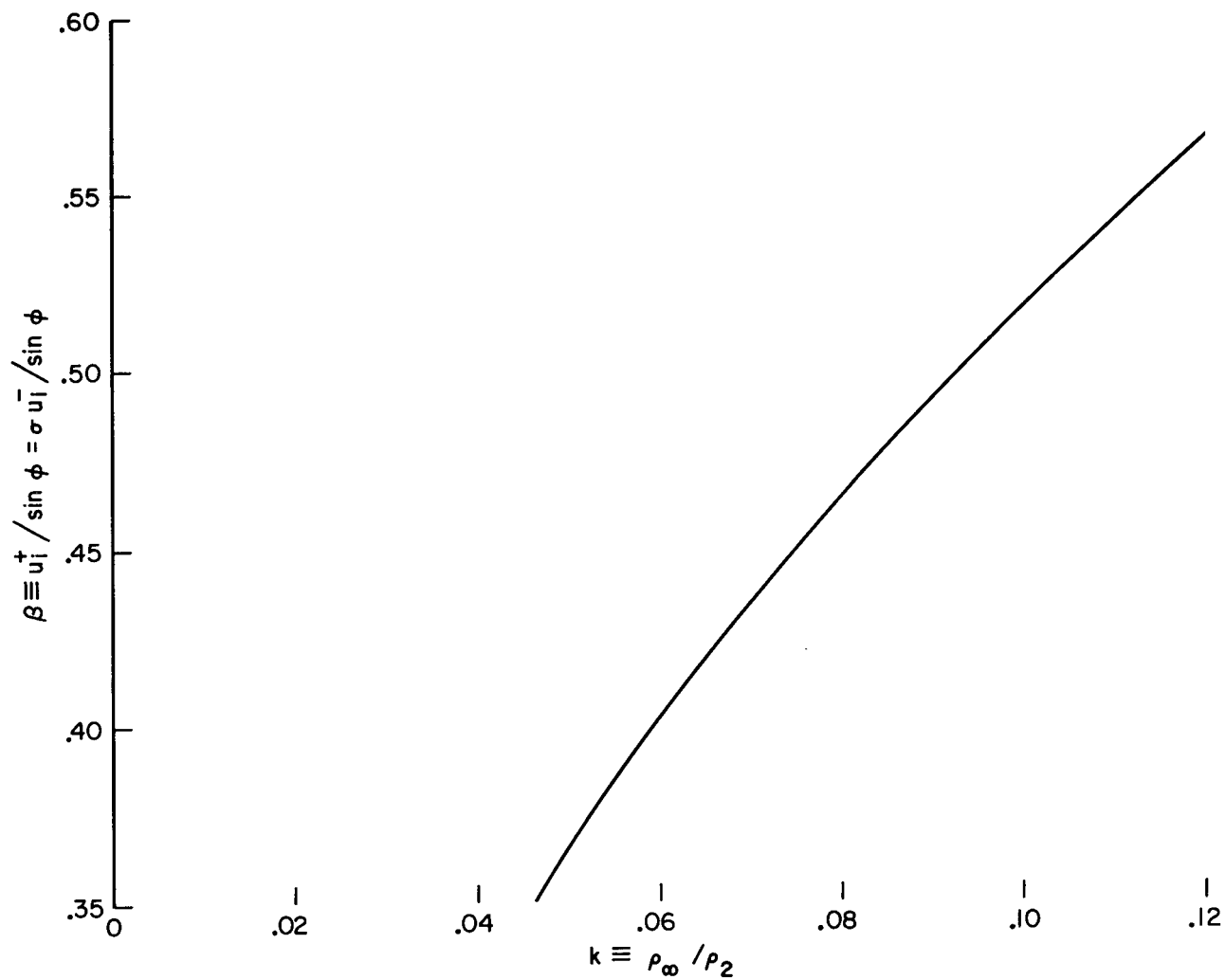
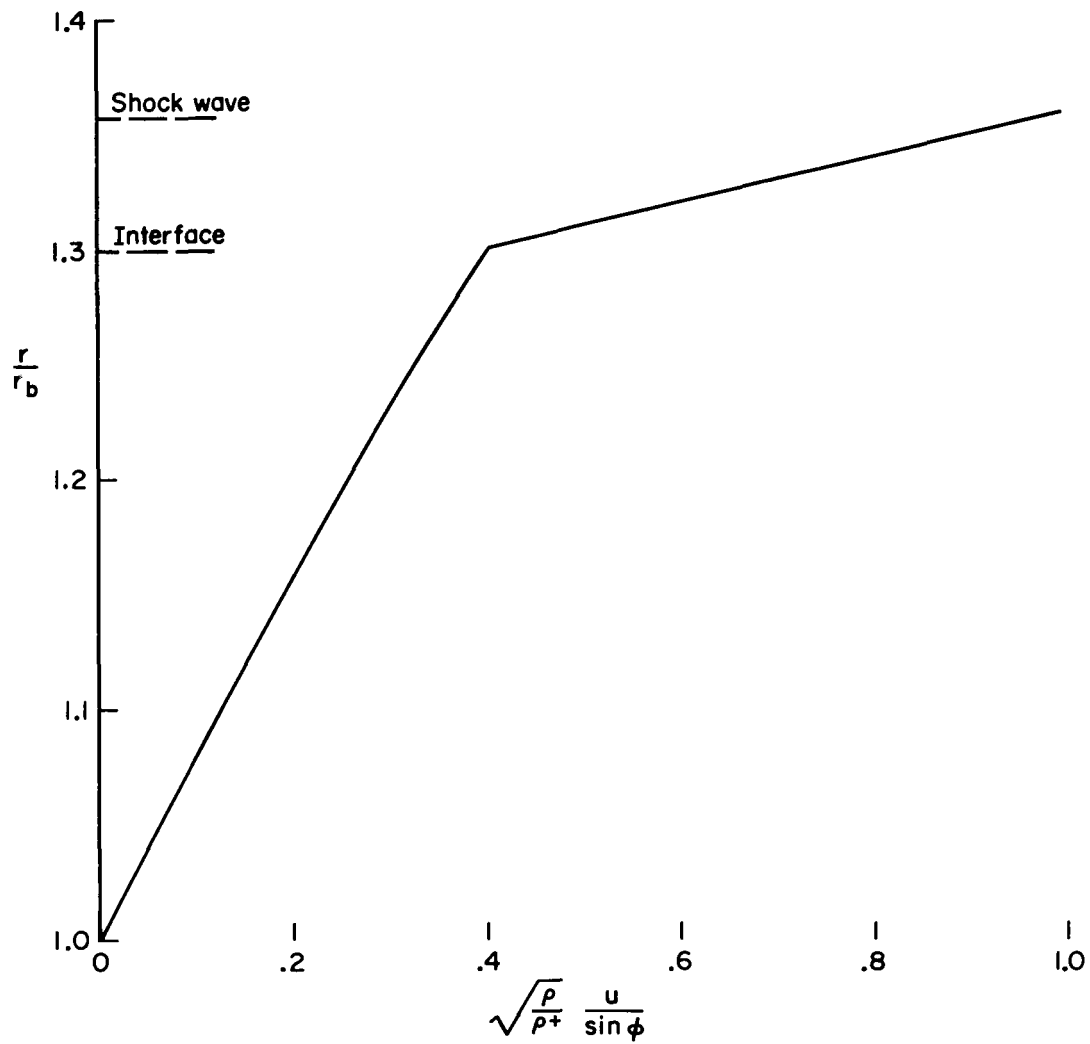


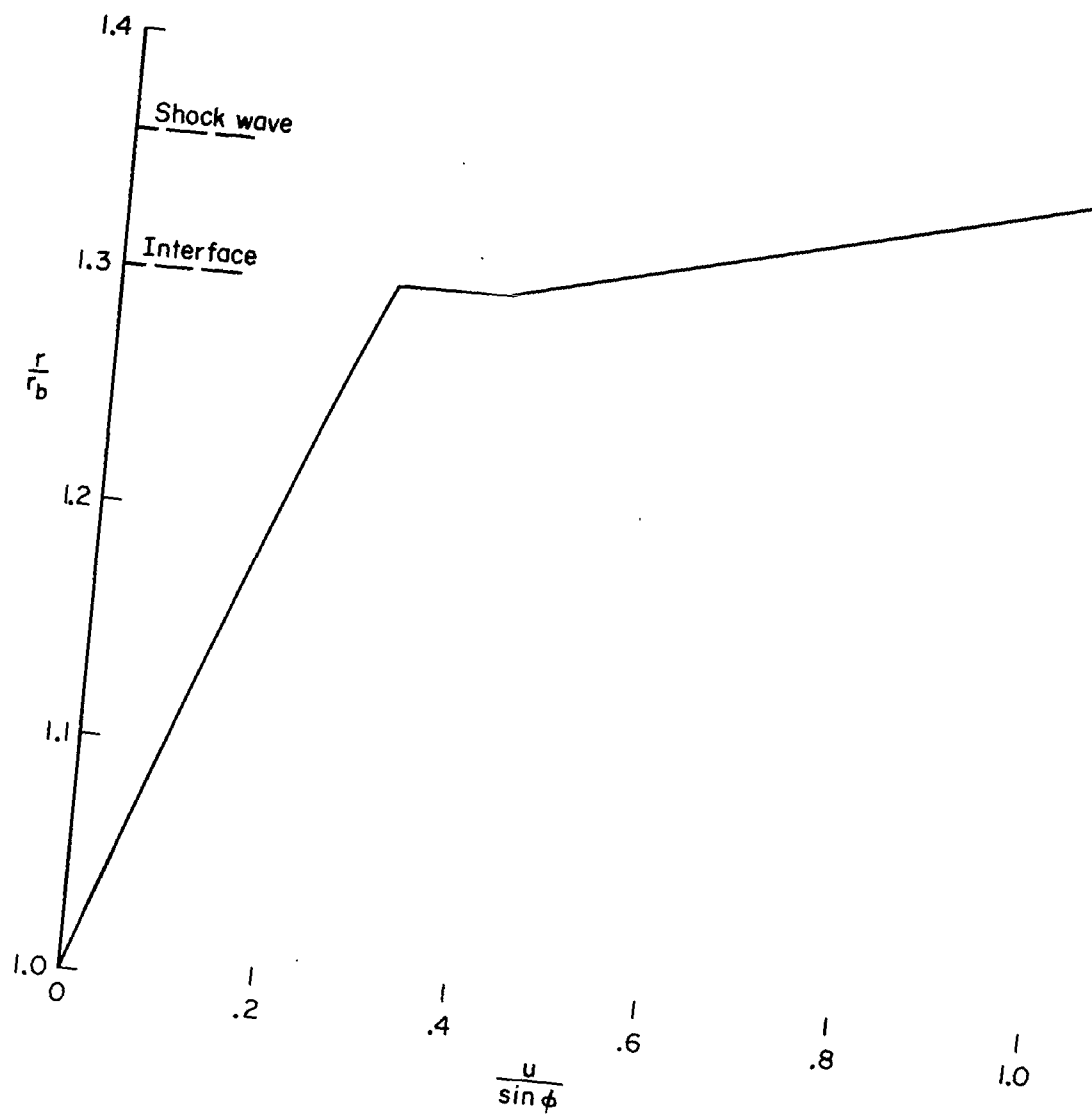
Figure 3.- Tangential velocity at the interface ( $v_\infty u_i$ ).



(a)  $k = 0.06$  ( $\beta = 0.402$ ),  $d_1/r_b = 0.3$  ( $\beta N = \sigma v_{e0} = 0.1395$ ), arbitrary  
 $\sigma^2 \equiv \rho^-/\rho^+$

Figure 4.- Examples of tangential-velocity profile for sphere.





(b)  $k = 0.06$  ( $\beta = 0.402$ ),  $d_1/r_b = 0.3$ ,  $\sigma^2 \equiv \rho^-/\rho^+ = 2$  ( $v_{eo} = 0.0985$ )

Figure 4.- Concluded.

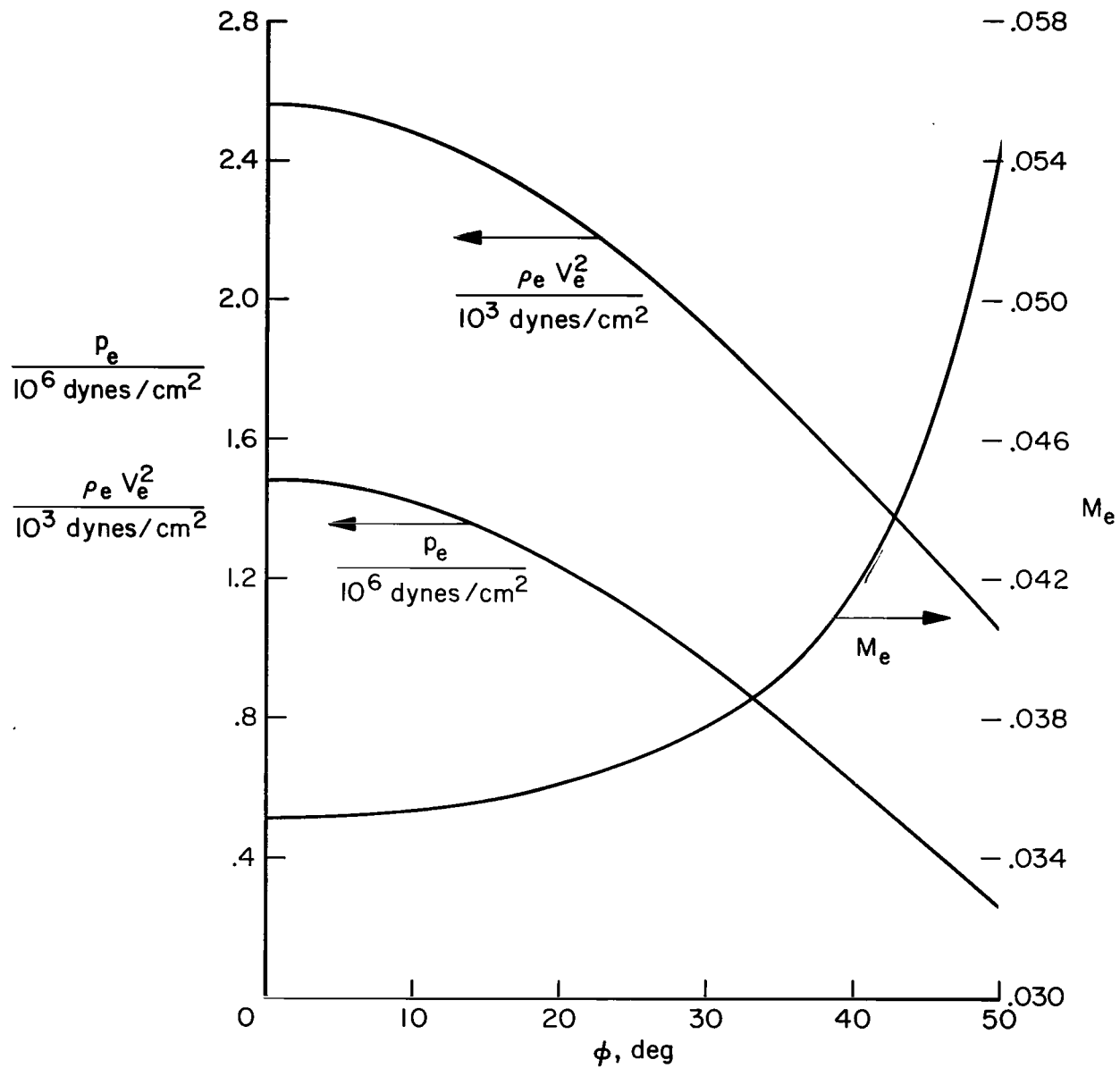


Figure 5.- Distribution along the boundary shock wave of flow parameters outside the boundary shock on a sphere at 60 km altitude;  $V_\infty = 20.83 \text{ km/sec}$  ( $k = 0.06554$ ) for  $N = 0.025$ ,  $\gamma = 7/5$ .

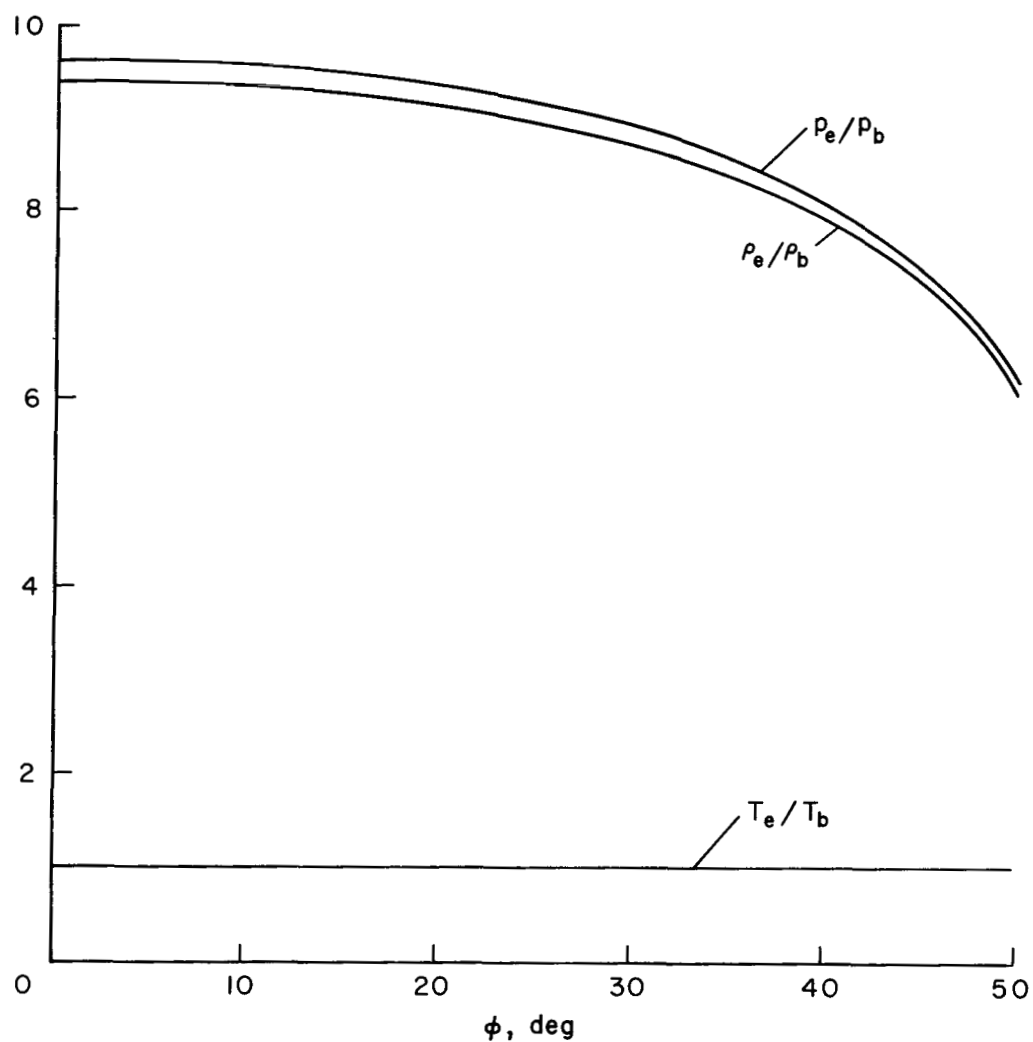


Figure 6.- Distributions along the boundary shock wave of conditions across the boundary shock on a sphere at 60 km altitude;  $V_\infty = 20.83$  km/sec ( $k = 0.06554$ ) for  $N = 0.025$  and for constant  $M_b = 1/3$  and  $\gamma = 7/5$ .

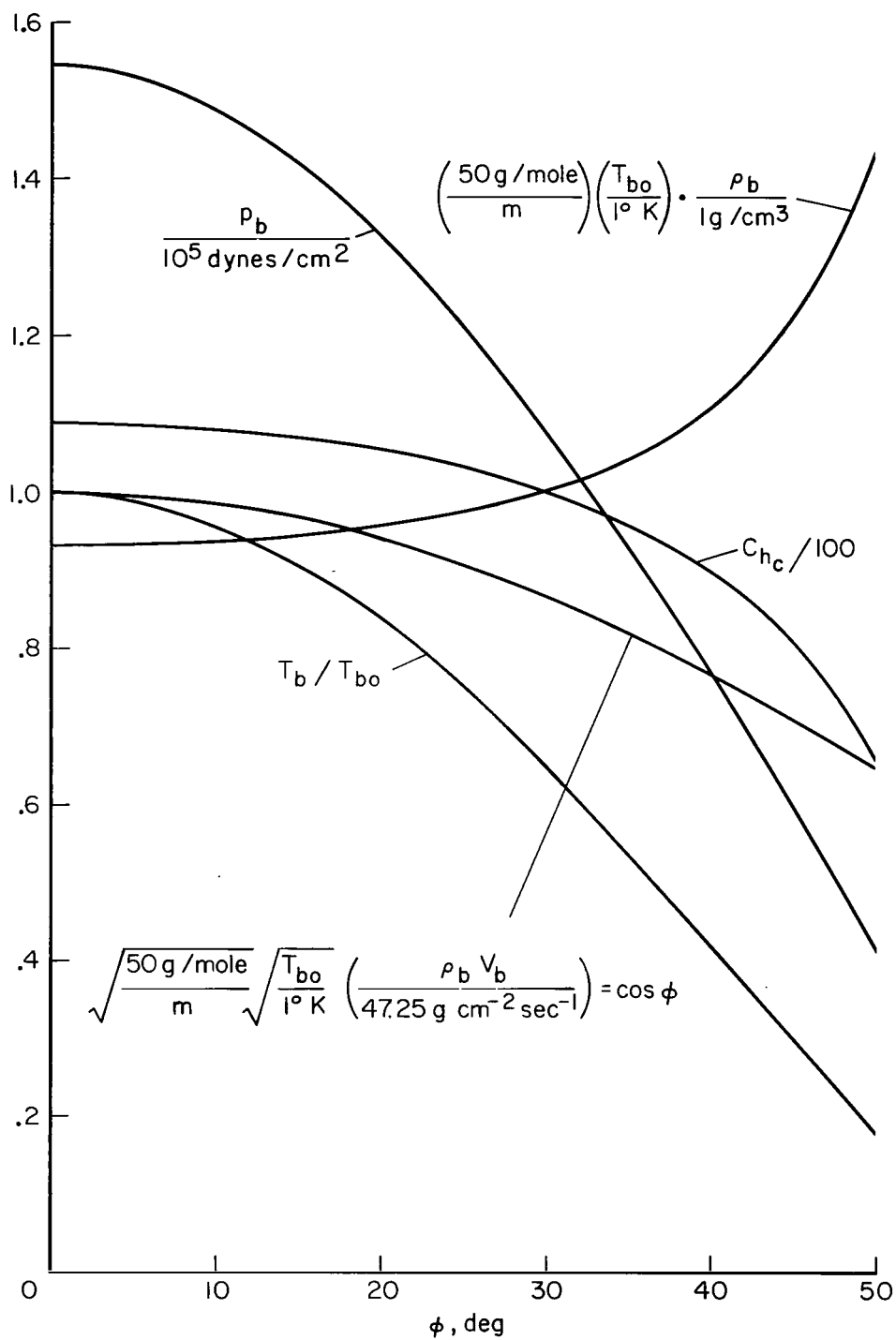


Figure 7.- Distributions along the boundary shock wave of conditions in the gas at the surface of a sphere at 60 km altitude;  $V_\infty = 20.83$  km/sec ( $k = 0.06554$ ) for  $N = 0.025$ ,  $M_b = 1/3$ ,  $\gamma = 7/5$ , and arbitrary  $T_{bo}$  and  $m$ .

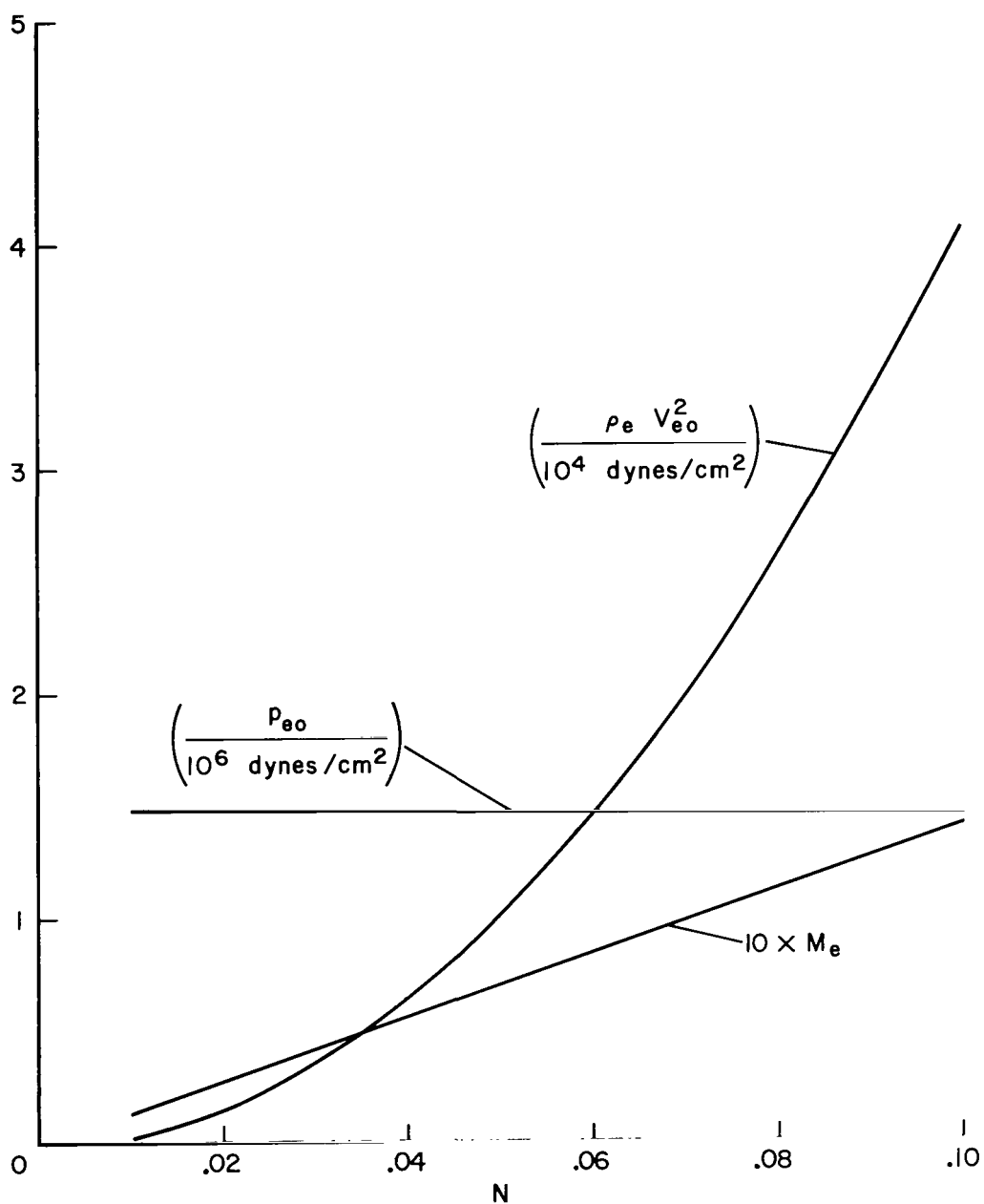
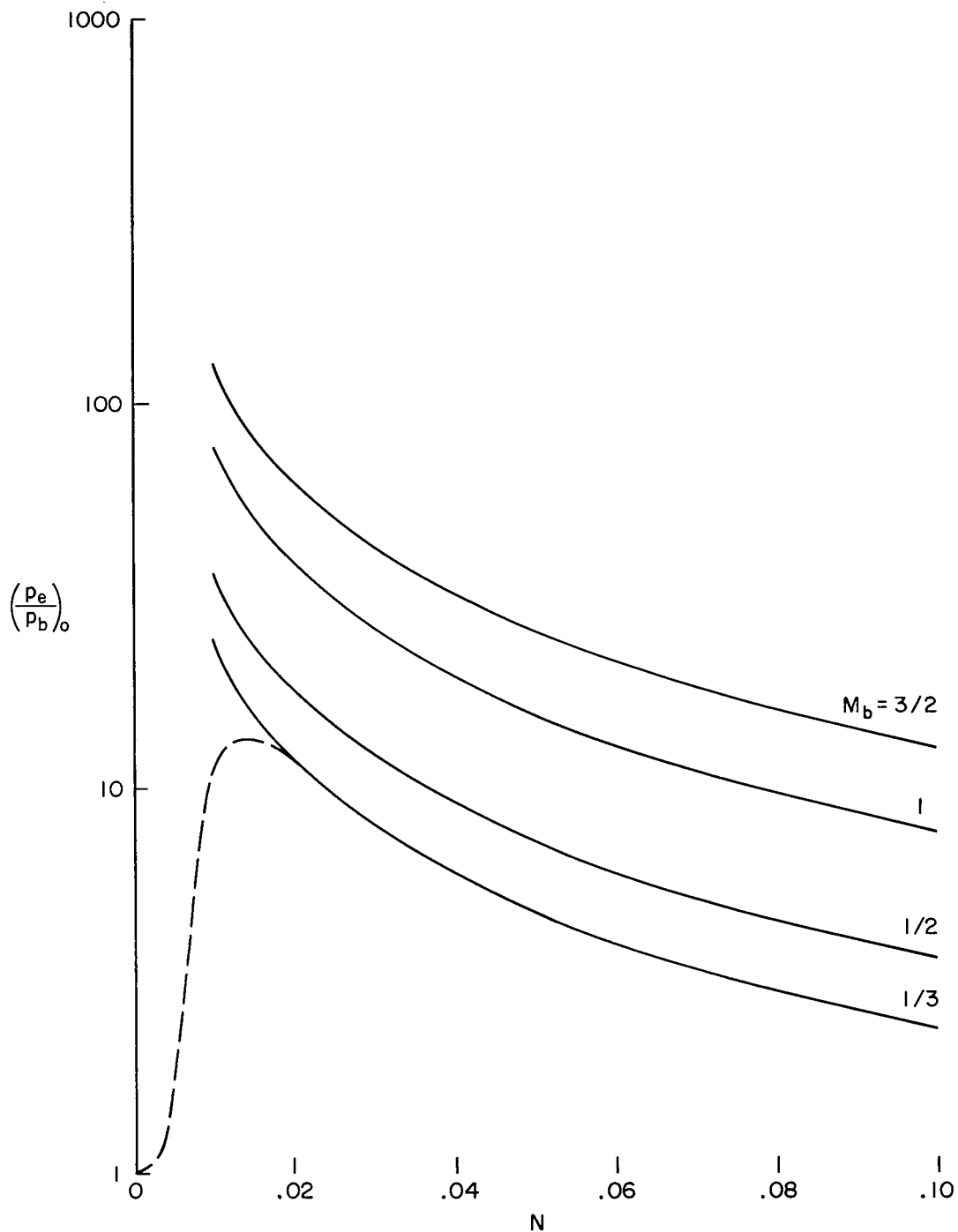
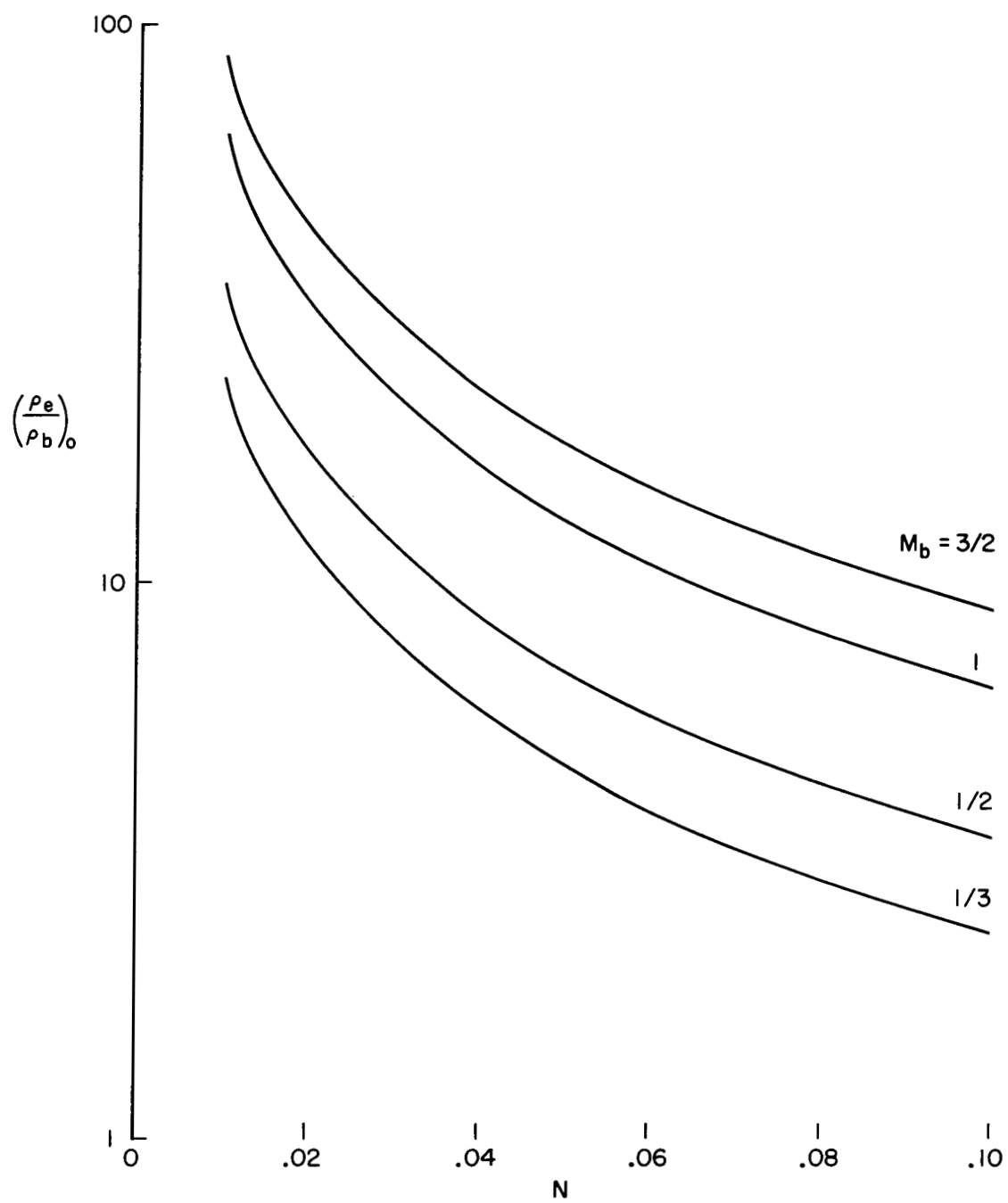


Figure 8.- Values (on the axis) of flow parameters outside the boundary shock wave on a sphere at 60 km altitude;  $V_\infty = 20.83$  km/sec ( $k = 0.06554$ ) for  $\gamma = 7/5$ .



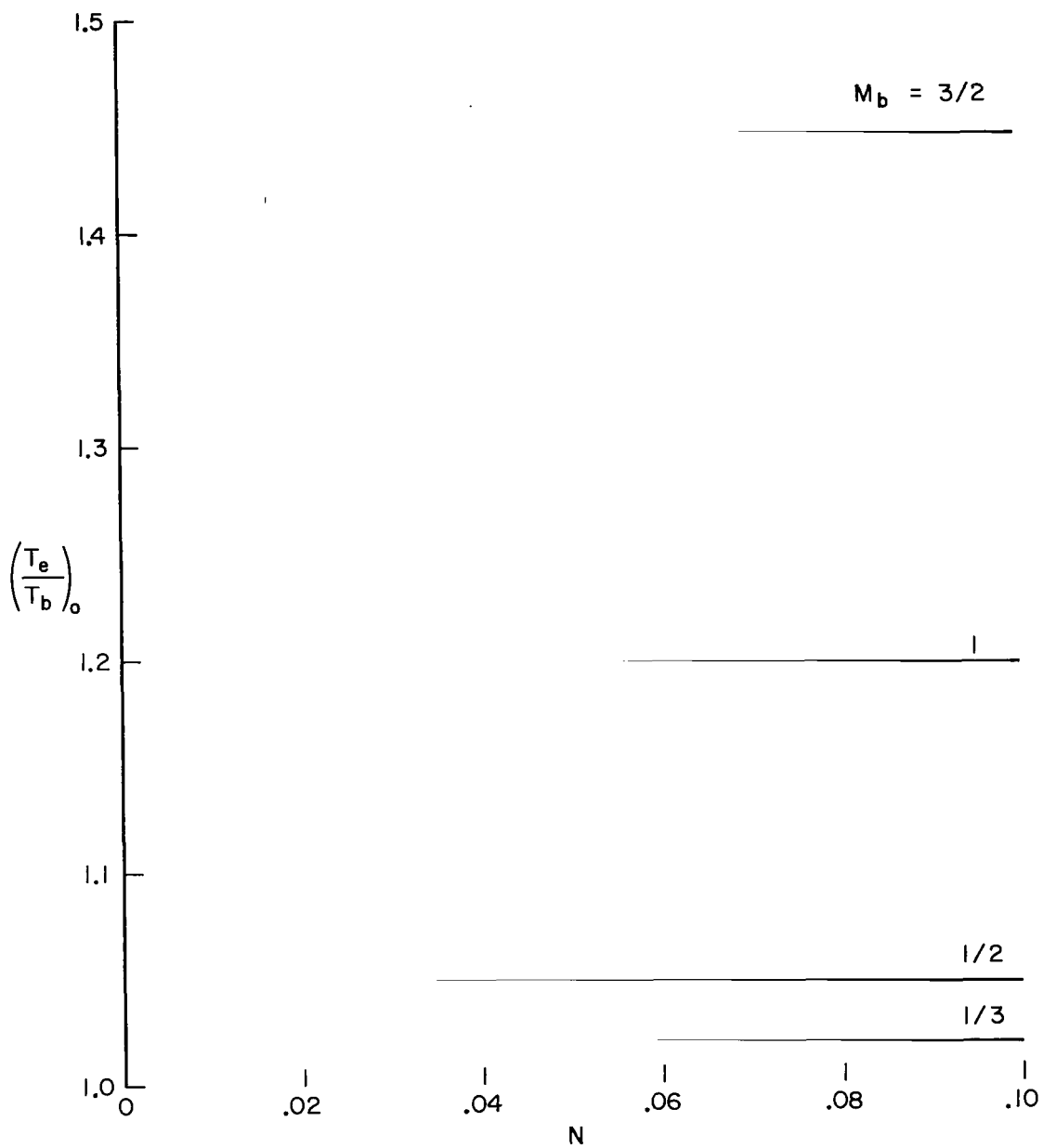
(a) Pressure ratio.

Figure 9.- Ratios of flow conditions across the boundary shock wave (at the axis) on a sphere at 60 km altitude;  $V_\infty = 20.83$  km/sec ( $k = 0.06554$ ) for  $\gamma = 7/5$ .



(b) Density ratio.

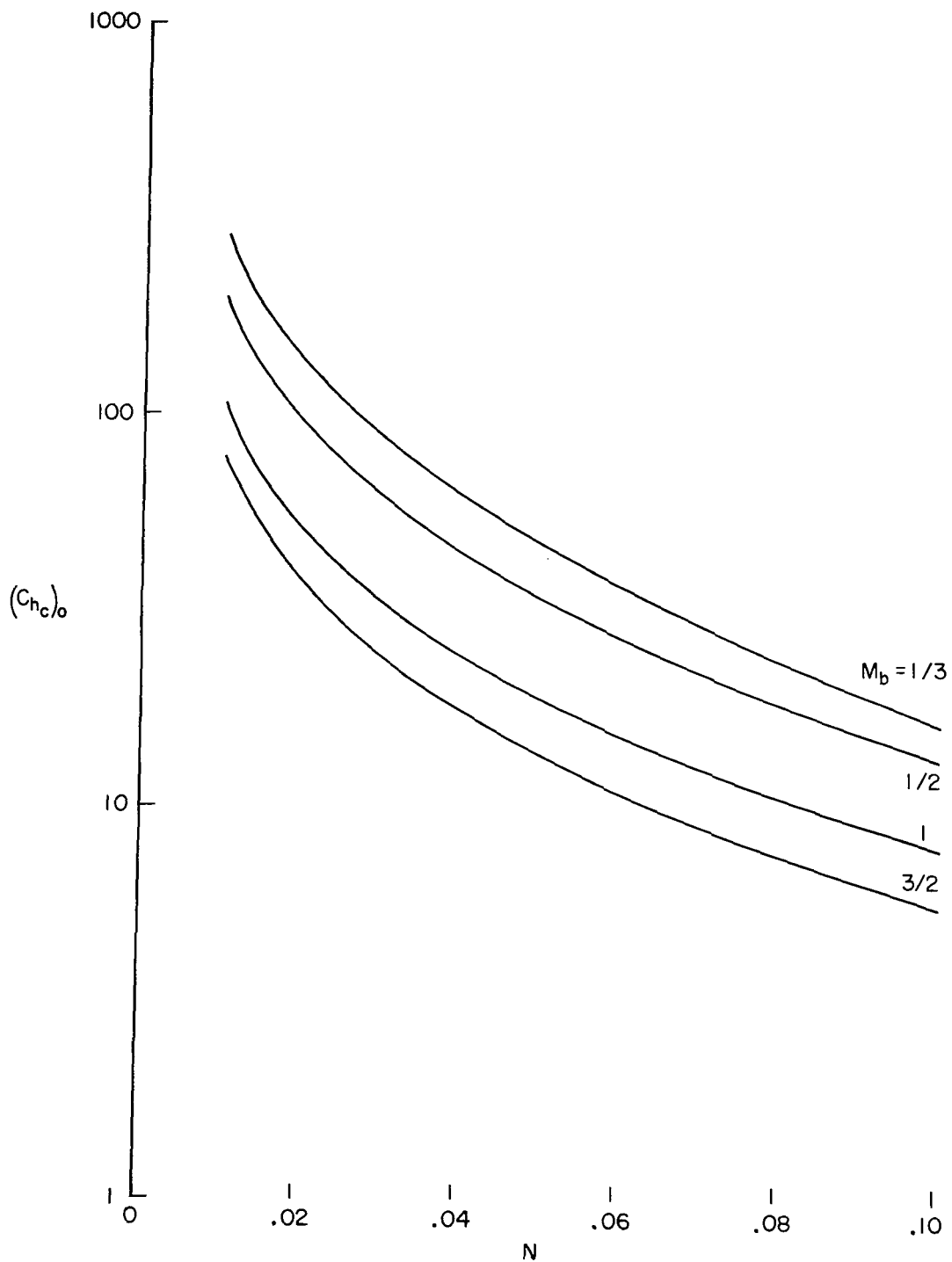
Figure 9.- Continued.



(c) Temperature ratio.

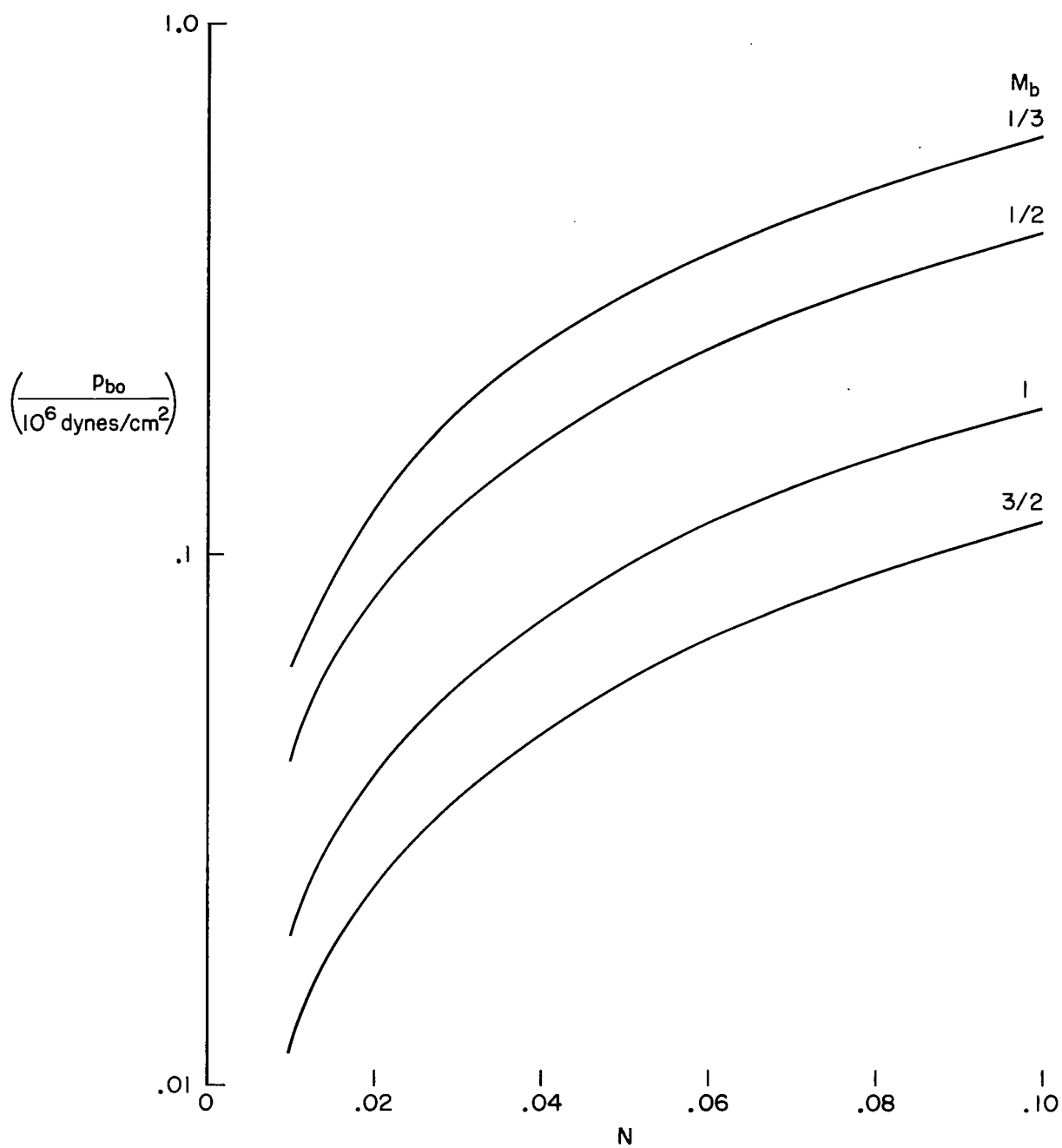
Figure 9.- Concluded.





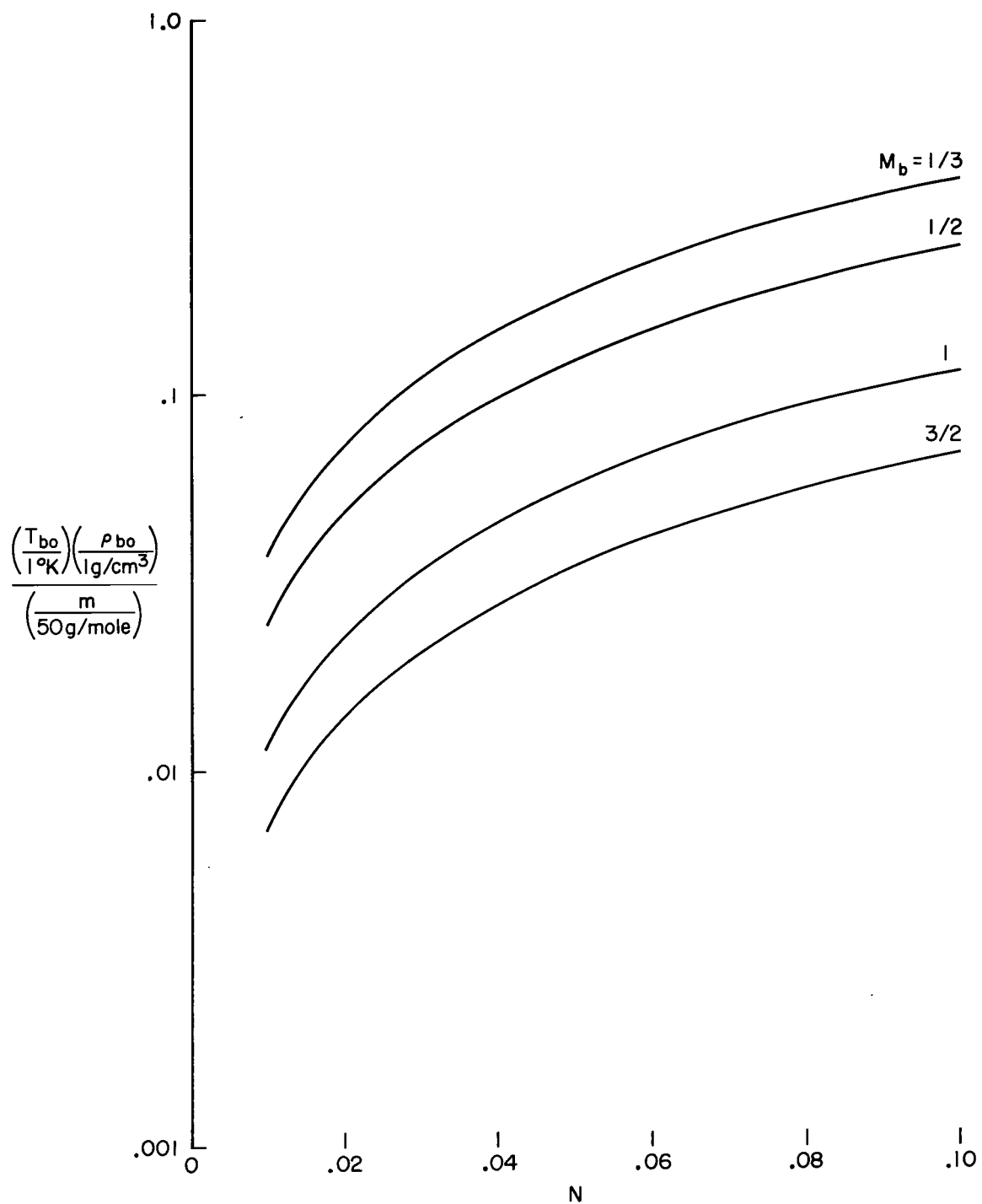
(a) Heat-conduction parameter.

Figure 10.- Values on the axis of flow parameters in the gas at the surface of a sphere at 60 km altitude;  $V_\infty = 20.83$  km/sec ( $k = 0.06554$ ) for  $\gamma = 7/5$ , various  $M_b$ , and arbitrary  $T_{b0}$  and  $m$ .



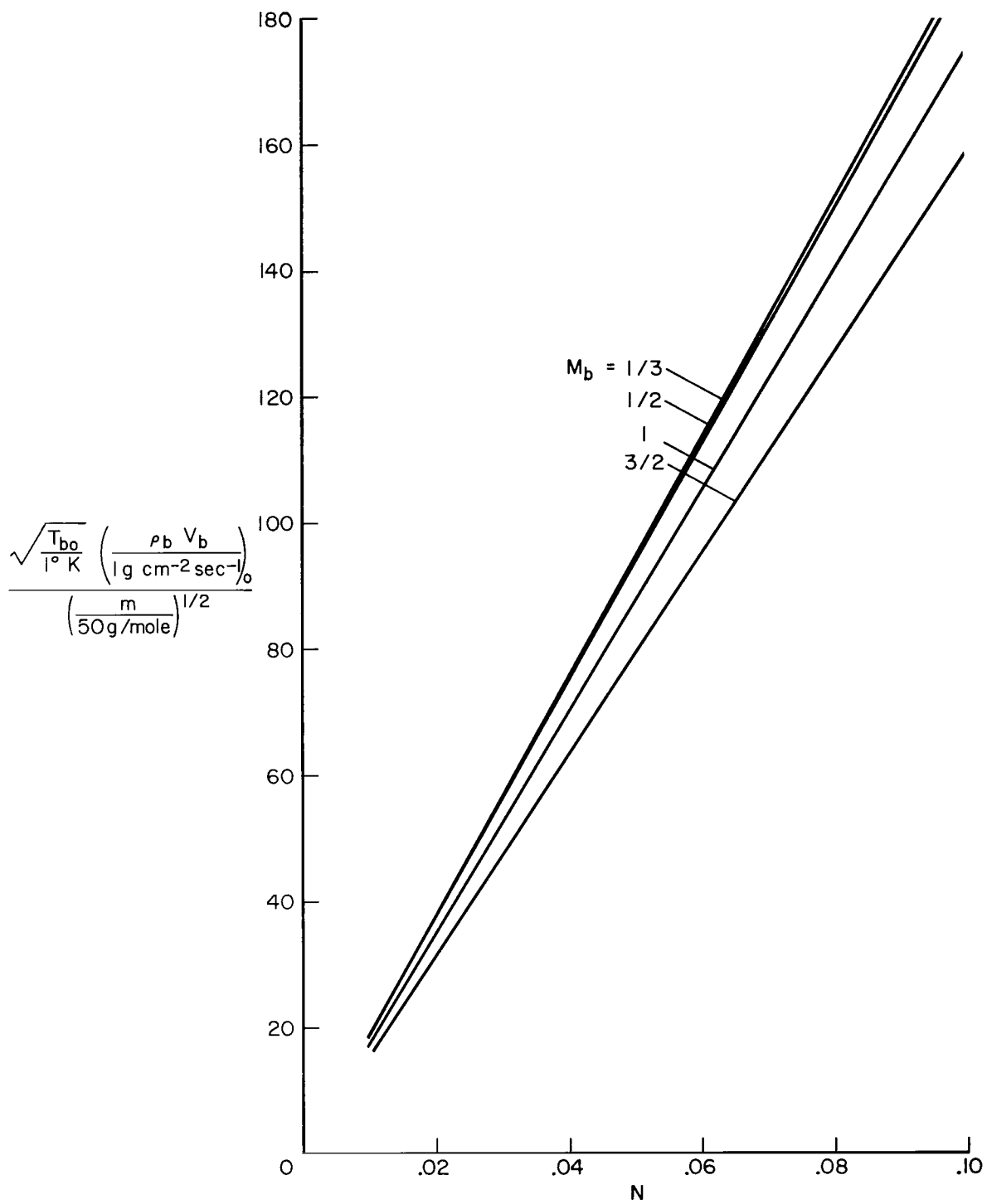
(b) Pressure.

Figure 10.- Continued.



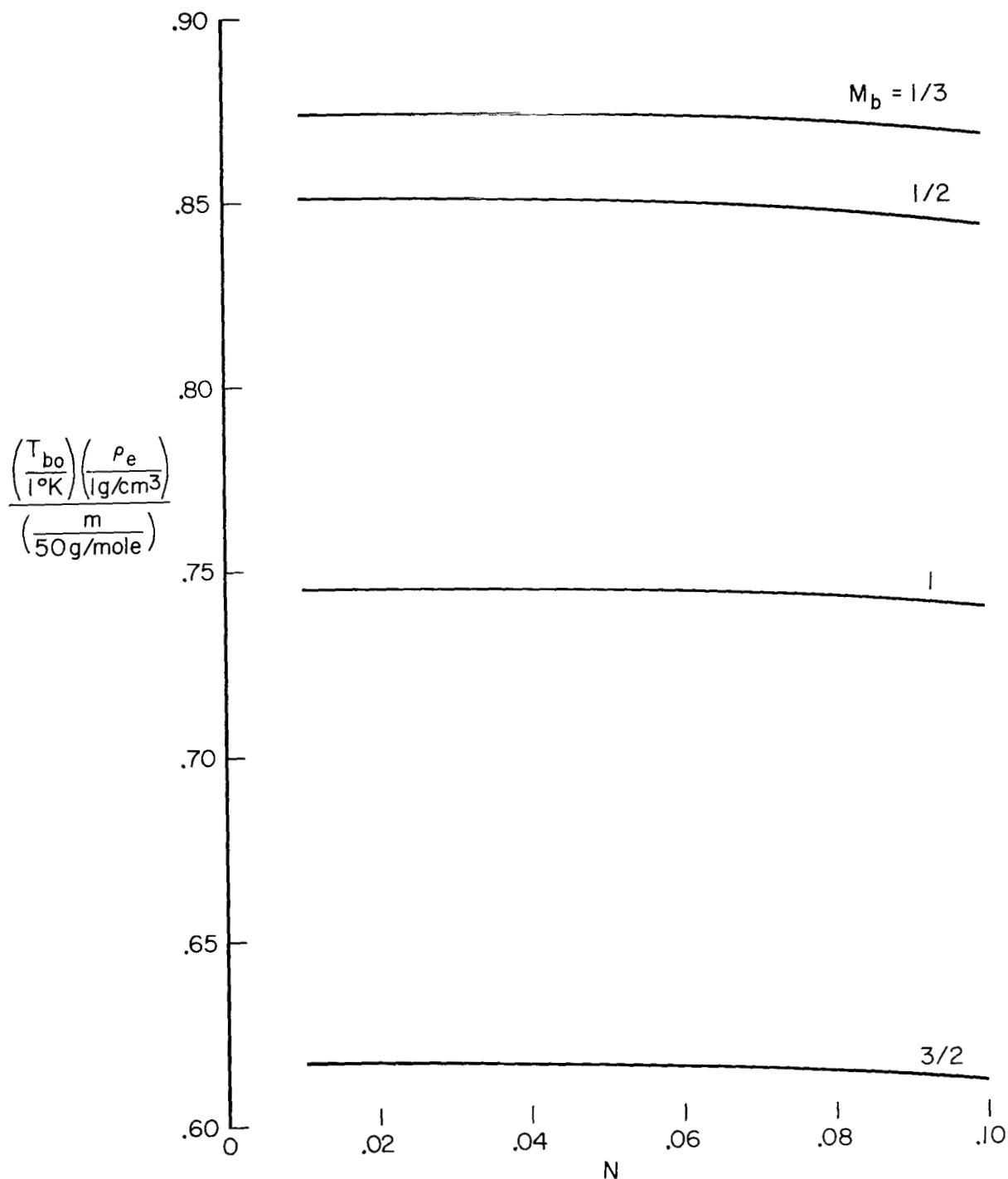
(c) Density (for given  $T_{bo}$  and  $m$ ).

Figure 10.- Continued.



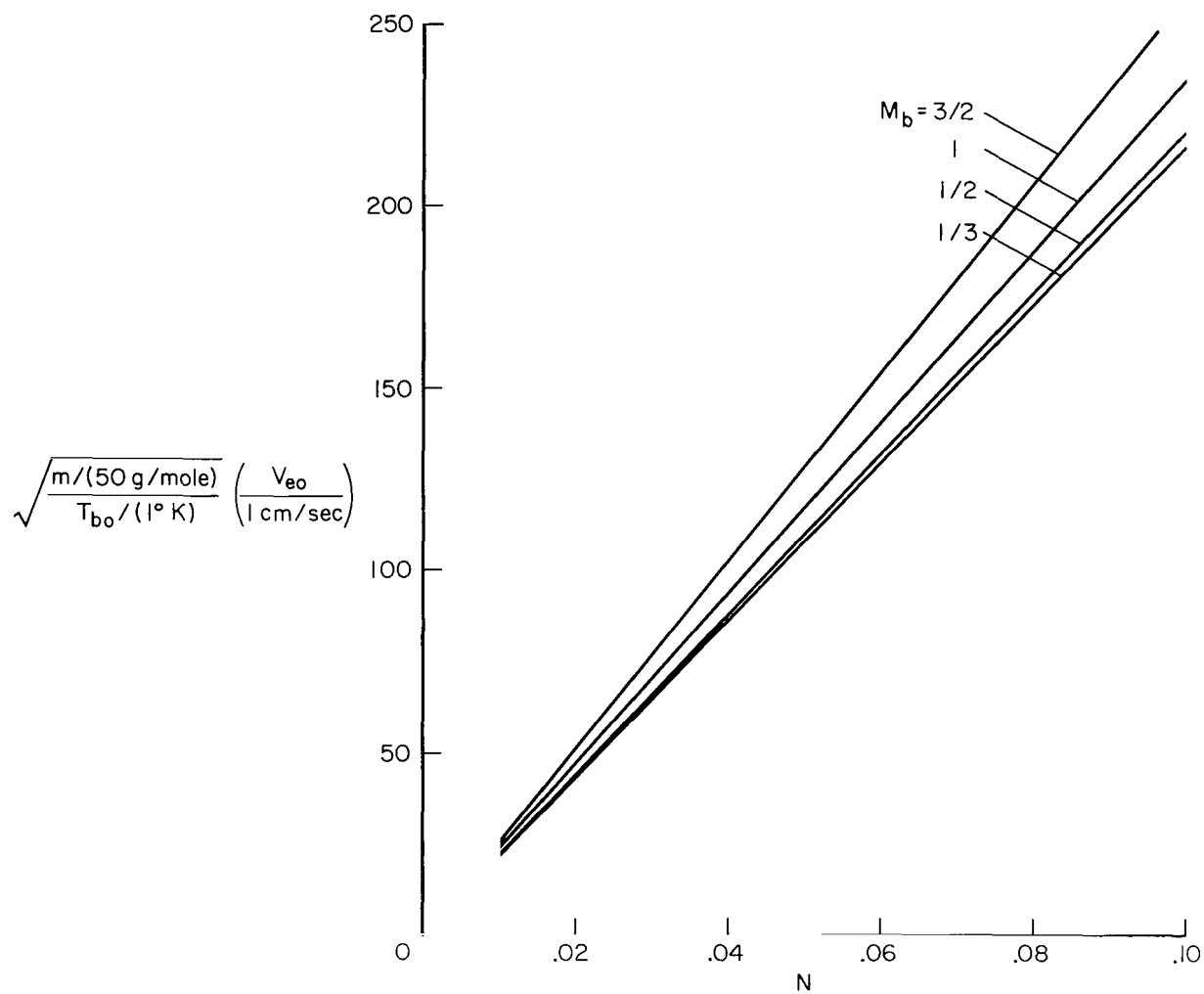
(d) Mass flux (for given  $T_{b0}$  and  $m$ ).

Figure 10.- Concluded.



(a) Density (for given  $T_{bo}$  and  $m$ ).

Figure 11.- Values (on the axis) of flow parameters outside the boundary shock wave on a sphere at 60 km altitude;  $V_\infty = 20.83$  km/sec ( $k = 0.06554$ ) for  $\gamma = 7/5$ , various  $M_b$ , and arbitrary  $T_{bo}$  and  $m$ .



(b) Velocity (for given  $T_{bo}$  and  $m$ ).

Figure 11.- Concluded.

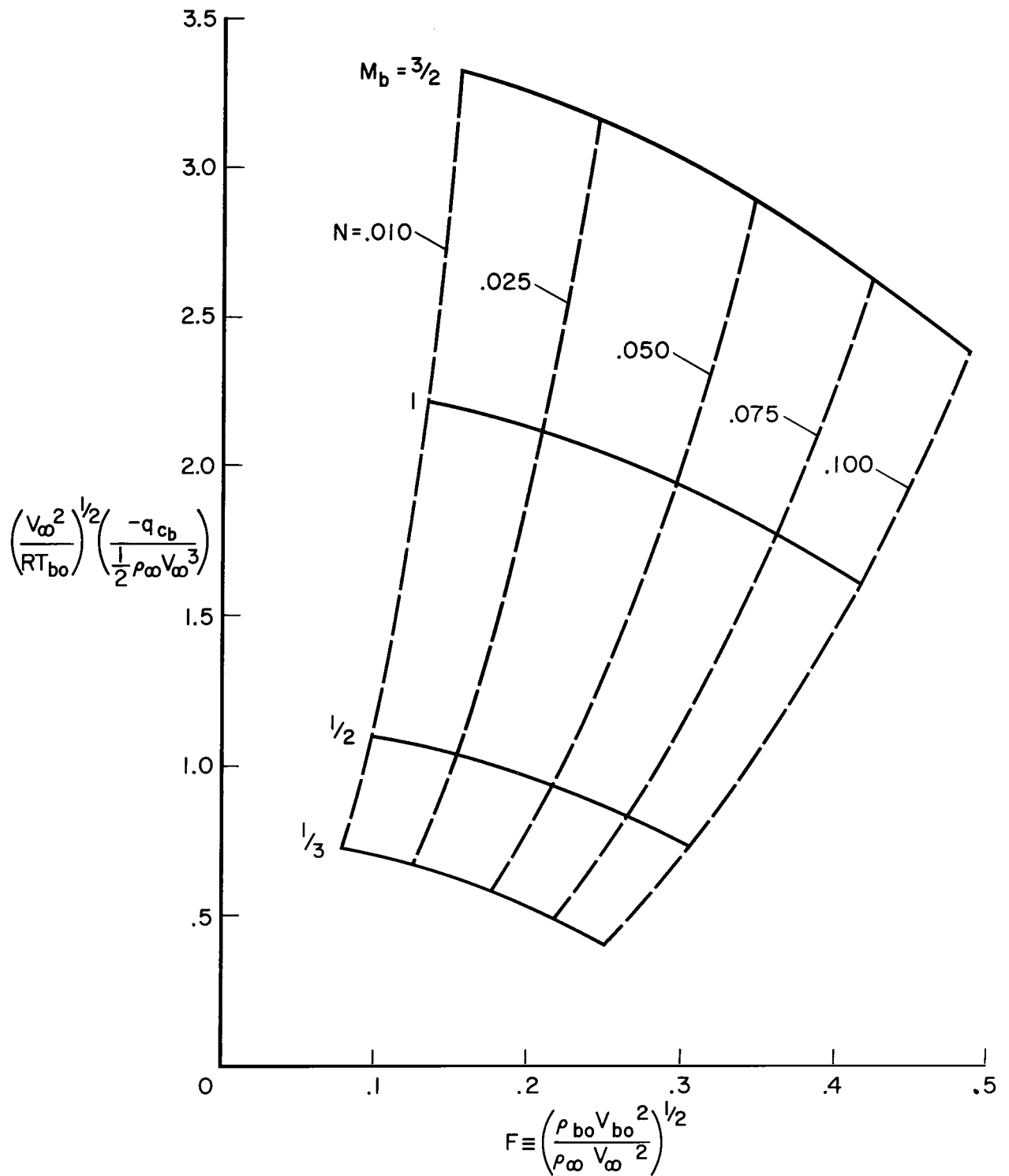


Figure 12.- Heat conduction versus blowing parameter for  $k = 0.06554$ ;  
 $\gamma = 7/5$ .

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